In the Proceedings of the International Conference on Biorheology, Sofia, Bulgaria, 18-22 October 2000, pp 67-71, 2000

A TRIPLE POROSITY MODEL OF STRESS INDUCED FLUID FLOW IN CORTICAL BONE

Solomon R. Pollack[1], Nikola Petrov[2],
[1] Department of Bioengineering, University of Pennsylvania, PA 19104-6392, USA, spollack@seas.upenn.edu
[2] Institute of Mechanics and Biomechanics, Bulgarian Academy of Sciences, 1113 Sofia, BULGARIA, petrov333@gmail.com

Abstract: ***A theoretical model to characterize the stress induced fluid flow in cortical bone has been developed using triple porosity theory. The construction of the model starts with assumption involving three interacting fluid compartments: microporosity of the bone matrix, the lacunar-canalicular porosity and the vascular porosity associated with the Haversian Canals. On the basis of deductive considerations and using available experimental and anatomical data, the model is consistently specified. The model is able to account for the observations of macroscopic streaming potentials at cyclic and step loading. The predicted results are: independent of specimen thickness as experimentally measured by Otter et al., (1994); and are characterized by two relaxation times that are present in the data of Pienkowski (1982), and Gross and Williams (1982).***

INTRODUCTION

In order to describe the macroscopic observations of streaming potentials, Salzstein et al., [1] developed two phase model of stress induced fluid flow in bone. The formulation was successful in modeling the frequency dependence of the streaming potential amplitude and phase relative to load. This model however is in conflict with later experimental studies of Otter et al., [2] and MacGinitie et al., [3]. They experimentally observed that decreasing the thickness of the specimen does not alter the effective time constant for the flow. Otter explains the observation with the assumption that in multi-osteonal specimens, the principle fluid relaxation occurs to Haversian Canals rather than to the bone surface. The aim of the present study is to offer macroscopic model of the stress induced fluid flow in bone, congruent with all existing macroscopic type experimental data at the present time. Our strategy is to start from triple porosity continuum model description of cortical bone and on the basis of existing information about the structure of bone, and applicable experimental data, to simplify and specify the model.

TRIPLE POROSITY MATHEMATICAL MODEL

Considering bone as a triple porosity deformed body, we model the bone utilizing multiple porosity consolidation theory (Wilson and Aifantis, [4], Cowin, [5]). Previously (Petrov et al., [6]), applying single porosity consolidation theory for an osteon, we assumed material isotropy and that the fluid pressure influence on the deformation state of the matrix is small. Using the same assumptions we have the following mathematical formulation for the problem:

$$\alpha^{mp}\frac{\partial}{\partial t}P^{mp} = k^{mp}\,div\,grad\,P^{mp} + \chi(P^{cl}-P^{mp}) + \lambda(P^{va}-P^{mp}) + \delta^{mp}\frac{\partial}{\partial t}e$$

$$(1)\quad \alpha^{cl}\frac{\partial}{\partial t}P^{cl} = k^{cl}\,div\,grad\,P^{cl} - \chi(P^{cl}-P^{mp}) + \gamma(P^{va}-P^{cl}) + \delta^{cl}\frac{\partial}{\partial t}e \quad ,$$

$$\alpha^{va}\frac{\partial}{\partial t}P^{va} = k^{va}\,div\,grad\,P^{va} - \lambda(P^{va}-P^{mp}) - \gamma(P^{va}-P^{cl}) + \delta^{va}\frac{\partial}{\partial t}e$$

where P^{mp}, P^{cl}, P^{va} are correspondingly the pressures in the microporosity, the canalicular-lacunar porosity and the vascular porosity and $\alpha^{(*)}, \delta^{(*)}, \chi, \gamma, \lambda, k^{mp}, k^{cl}, k^{va}$ are rheological constants. The term e is the trace of strain tensor, which is considered as known function of time and position, and which corresponds to small coupling between the fluid pressure and the deformation of the matrix. We note that the radii of microporosity, canalicular porosity and vascular porosity belong to different scales of size. Zhang et al., [7] suggest for these three levels of porosity permeability ratios of $1:10^2:10^6$. This causes the pressure within the vascular porosity to be practically arterial for in vivo experiments and atmospheric for in vitro experiments. To simplify the model we neglect the microporosity permeability as of orders of magnitude less than the permeability of canalicular-lacunar porosity. Taking into account the hierarchical structural organization of the bone, we note that the microporosity has a small interface with the vascular porosity compared to the interface between the microporosity and canalicular - lacunar porosity. Therefor the direct exchange of fluids between vascular porosity and microporosity can be neglected, and λ in equations (1) can be set to zero.

The bone samples used, in the study of Starkebaum et al., [8], Salzstein and Pollack [9], Otter et al., [10] and others, were subjected to four - point bending so that a state of pure bending was imposed. Accordingly only one space coordinate - y (the distance from the neutral middle specimen surface) is required. Setting the value for the pressure of the fluid within Haversian Canals equal to zero (ambient pressure) we obtain:

(2)
$$\frac{\partial}{\partial t}P^{cl} = D\frac{\partial^2}{\partial y^2}P^{cl} - \frac{\beta^c}{\tilde{\tau}^c}(P^{cl} - P^{mp}) - \frac{P^{cl}}{\tau^d},$$
$$\frac{\partial}{\partial t}P^{mp} = \frac{1}{\tilde{\tau}^c}(P^{cl} - P^{mp}) \quad , \quad t > 0$$

where

$$D = \frac{k^{cl}}{\alpha^{cl}}, \ \tilde{\tau}^c = \frac{\alpha^{mp}}{\chi}, \ \tau^d = \frac{\alpha^{cl}}{\gamma}, \ \beta^c = \frac{\alpha^{mp}}{\alpha^{cl}}, \ \vartheta^{cl} = \frac{\delta^{cl}}{\alpha^{cl}}, \ \vartheta^{mp} = \frac{\delta^{mp}}{\alpha^{mp}}.$$

The boundary conditions which must be satisfied are the zero pressure at the neutral surface and zero gradient of pressure over the outer surface (i.e. fluid does not leave the specimen):

(3) $$P^{cl}(0,t) = 0, \ \frac{\partial}{\partial y}P^{cl}(h,t) = 0,$$

where h is half of the thickness of the sample. Because of the presence in Eqn. (2) of the second partial derivative of the fluid pressure the solutions of the corresponding boundary problem depend on the sample thickness. The only way to reconcile the triple porosity consolidation theory with the Otter's experiments is to assume that the field term in Eqn. (2) - $D\frac{\partial^2}{\partial y^2}P^{cl}$ is negligibly small compare to the other terms. This assumption means that dominant drainage of the fluid is in Haversian Canals and leads to the following particular solutions:

(A). case of canalicular-lacunat pressure relaxation after step loading of the specimen-

(4) $$P^{cl}(y,t) = P^0(y)\left(\frac{(\tilde{\tau}^c - \tau_1)}{\tau_2 - \tau_1}\exp(-\frac{t}{\tau_1}) + \frac{(\tau_2 - \tilde{\tau}^c)}{\tau_2 - \tau_1}\exp(-\frac{t}{\tau_2})\right),$$

where $P^0(y) = P^{cl}(y,0)$ and

(5) $$\frac{1}{\tau_{1,2}} = \frac{1}{2}\left(\frac{1+\beta^c}{\tilde{\tau}^c} + \frac{1}{\tau^d}\right) \pm \sqrt{\frac{1}{4}\left(\frac{1+\beta^c}{\tilde{\tau}^c} + \frac{1}{\tau^d}\right)^2 - \frac{1}{\tilde{\tau}^c\tau^d}};$$

(B). case of complex amplitude of canalicular-lacunar pressure at stationary cyclic loading of the specimen-

(6) $$\overline{P}^{cl}(y)=\frac{\left(\omega^2\vartheta^{cl}-i\omega\dfrac{\vartheta^{cl}+\beta^c\vartheta^{mp}}{\widetilde{\tau}^c}\right)}{\left(\omega^2-\left(\dfrac{1+\beta^c}{\widetilde{\tau}^c}+\dfrac{1}{\tau^d}\right)i\omega-\dfrac{1}{\widetilde{\tau}^c\tau^d}\right)}\frac{3\mu_1}{\mu_1+K_1}\frac{y}{R^b_{\min}},$$

where $i,\omega,\ \mu_1, K_1, R^b_{\min}$ are imaginary unite, angular velocity, shear modulus of bone matrix, bulk modulus of bone matrix and minimal bending radius of the sample respectively.

The ability of the theory to fit the experimental data is represented on Fig. 1 for step loading type data obtained by Otter et al., [2], and on Fig. 2 for cyclic loading type data obtained by Salzstein and Pollack [9].

DISCUSSION AND CONCLUDING REMARKS

A macroscopic model for stress generated fluid flow in cortical bone has been formulated, treating bone as a triple porosity media in which the hierarchical organization of the bone fluid paths have been taken into account. The model is in fundamental contradiction with the homogeneous models of Salzstein et al., [1] and, Weinbaum et al., [11] in which the drainage within the Haversian Canals is not considered. Even within the limitations of a homogeneous model, the present new model implicitly, by means of interaction between the canalicular - lacunar porosity and vascular porosity, accounts for the drainage functions of the Haversian Canals.

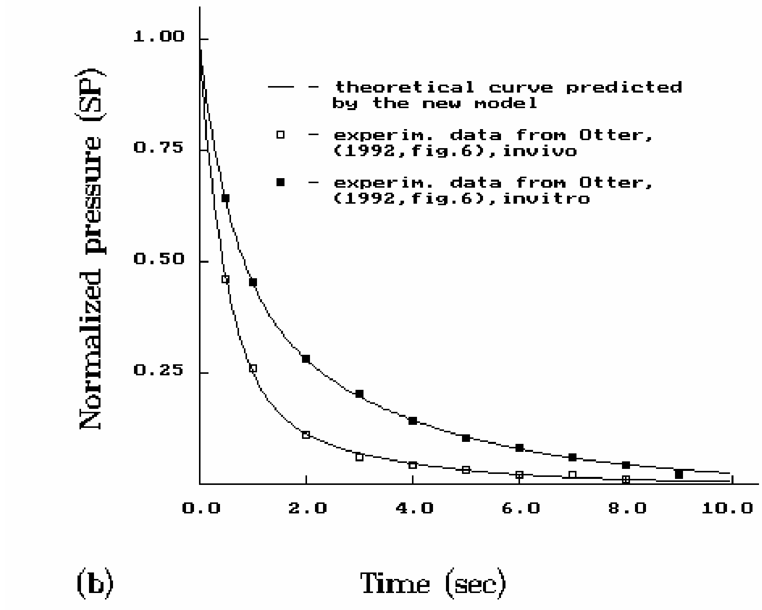

Figure 1: Pressure relaxation after step loading (in vitro- $\tau^d=1.0$, $\beta^c=1.0$, $\widetilde{\tau}^c=2.0$; in vivo- $\tau^d=0.6$, $\beta^c=0.6$, $\widetilde{\tau}^c=2.1$)

Opposite to our earlier study (Salzstein et al., [1]) and the study of Weinbaum et al., [11], and Cowin et al., [12] the stress generated macroscopic pressure difference is not the result of transcortical convective flow. Our assumption for the existence of free fluid in the microporous space, exchangeable with canalicular - lacunar porosity, is in conflict with the hypothesis of fully immobile microporosity water of Weinbaum et al., [11] and Zhang et al., [7]. At the end we can

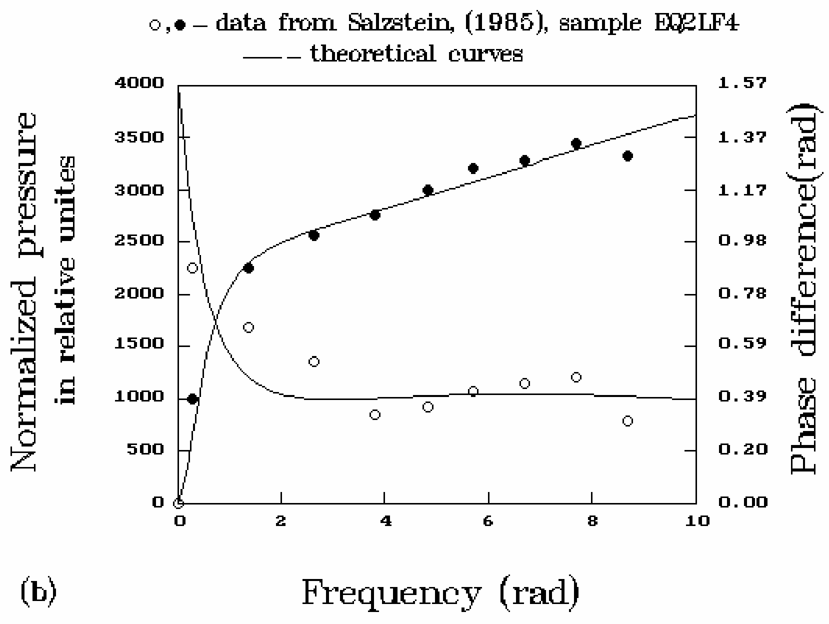

Figure 2: Amplitude and phase diagrams at cyclic loading ($\tau^d = 0.4$, $\beta^c = 3.2$, $\tilde{\tau}^c = 0.42$)

conclude that this new triple porosity theoretical model, presents for the first time, the possibility to explain all of the existing macroscopic type experimental streaming potential data.

REFERENCES

[1] R. A. Salzstein, S. R. Pollack, A. F. Mark and N. Petrov, "Electromechanical potentials is cortical bone- I. A continuum approach", *J. Biomechanics ,* vol. 20, pp. 261-270, 1987.

[2] M. W. Otter, L. A. MacGinitie, K. G. Seiz, M. W. Johnson, R. B. Dell and G.V. B. Cochran, "Dependence of streaming potential frequency response on sample thickness: implication for fluid flow through bone microstructure", *Biomemetics ,* vol. 2, pp.57 -75, 1994.

[3] L. A. MacGinitie, K. G., Seiz, M. W. Otter and G. V. B. Cochran, "Streaming potential measurements at low concentrations reflect bone microstructure", *J. Biomechanics,* vol. 27, pp., 969-978, 1994.

[4] R. K. Wilson and E. Aifantis, "On the theory of consolidation with double porosity", *Int. J. Engng Sci.*, vol. 20, pp. 1009-1035, 1982.

[5] S. C. Cowin, "Bone poroelastisity", *J. Biomechanics,* vol. 32, pp. 217-238,1999.

[6] N. Petrov, S. R. Pollack and R. Blagoeva, "A discrete model for streaming potentials in single osteon", *J. Biomechanics*, vol. 22, pp. 517- 521,1989.

[7] D. Zhang, S. Weinbaum, and S. C. Cowin, (1998) On the calculation of bone pore water pressure due to mechanical loading. *Int. J Solids Struct.* 35, 4881-4897, 1998.

[8] W. S. Starkebaum, S. R. Pollack and E. Korostoff, "Microelectrode studies of stress-generated potentials in four- point bending of bone", *J. Biomed. Mat. Res.*, vol.13, pp. 729-751, 1979.

[9] R. A. Salzstein and S. R. Pollack, "Electromechanical potentials is cortical bone- II. Experimental analysis", *J. Biomechanics*, vol. 20, pp. 271-280, 1987.

[10] M. W. Otter, V. R. Palmieri, D. D. Wu, K. G. Seiz, L. A. Macginitie and G. V. B. Cochran, "A comperative analysis of streaming potentials in vivo and invitro", *J. Orthop. Res.*, vol. 10, pp. 710-719, 1992.

[11] S. Weinbaum, S. C. Cowin and Y. Zeng, "A Model for the excitation of osteocytes by mechanical loading- induced bone fluid shear stresses", *J. Biomechanics*, vol. 27, pp. 339-360, 1994.