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Dry Bone as a Quadrupole Piezodielectric

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In spite of the considerable quantity of available experimental data there exists no conformity about the structural factors responsible for the piezoelectric properties of bone. The only indisputable fact is that the piezoelectric behaviour of bone is determined by its inorganic part, i. e. by its extracellular matrix. In their initial experiments Fukada and Yasuda [1] use samples air-dried for years. Cochran [2] uses samples which are subjected to different procedures — temperature and chemical treatment included. The comparison between samples subjected to different treatments and such just taken from a corpse shows that no significant differences are registered. Analogous results have been obtained by other authors as well.

Fukada and Yasuda [1] accept that the piezoelectric properties of the bone tissue are due to the helic configuration of the collagen fibres. Later on Fukada and Takashita [3], Cohen and Edelman [4] point out that it is quite possible the stress-induced reorientation of dipoles in polypeptide units resulting from mechanical distortion of the molecules to be responsible for bone polarization.

Becker, Bassett and Bachman [5] consider the semi-conductivity of the bone tissue to be one of the factors determining the stress-generated potentials. It is well known that at deformation the p-n junction between two semiconductors polarizes electrically. In bone such p-n junctions are realized between the collagen fibres which exhibit n-conductivity and by the appatite crystals exhibiting p-conductivity.

Anderson and Eriksson [6, 7] accept the existence of "streaming potentials" generated by the flux of ions across a mollecular surface of different charge. If this effect is dominating, it should be expected that with the increase of the frequency under periodic loading the generated potentials will decrease and at frequency higher than 1000 Hz they will disappear entirely [8]. But it is in contradiction with some experimental studies where the frequencies used are between 2000 and 4000 Hz [1, 9, 10].

The scientists have disagreements in principle concerning not only the microstructure factors, but also the phenomenologic model by which the electromechanical interaction in bone should be described. Bassett and Becker [11] suppose that the piezoelectric properties of bone are not in agreement with the classical piezoelectric theory. Their point of view is strongly criticized by Shamos and Lavine [12] who share the opinion that it is not necessary more complex models to be constructed for the explanation of the experimental data. Reinish and Nowick come to the same conclusion in their works [9, 10].

Williams and Breger [13] compare the potentials generated by some classic piezomaterials, dry bone and tendon. The beams of quartz, tourmaline, bone and tendon are turned end-for-end and remeasured. Both piezoelectric crystals gave the opposite sign of the transverse voltage on deflection, both bone and tendon gave the same sign. Williams and Breger point out that according to the classical theory the bone plates should not have to generate piezoelectric potentials at pure bending, while the experimental studies of Steinberg et al. [14] show generation of such potentials.

For the purpose of obtaining an appropriate analytical model Williams and Breger [13] propose an empirical equation in which the electric field depends not only on the stress but on the stress gradient as well. In the case of a homogeneous field of stress the model is in conformity with the classical piezoelectric theory. It should be noted that the scientists who accept the validity of the classical theory usually base their views on experiments in which the stress field is homogeneous.

In the present papers the authors show that the results following from the empiric model proposed by Williams and Breger are in a consequence with the quadrupole piezoelectric theory purposed by Brankov and Petrov [15]. The considerations concern dry bone which, as it is well known, exhibits high electrical resistance and could be accepted as an ideal dielectric.

Quadrupole Model

The relationship between bone structure and induced electric potentials is very successfully discussed by McElhaney [16] who has carried out his experiments on a whole bone. The structure of the bone tissue is far more complex than of the classical piezomaterials. That is why for the modelling of the piezoelectric potentials of bone a more precise accounting of its electric structure is needed. This can be done by the quadrupole theory in which the quadrupole moment should be taken into account as well as the dipole moment presented by polarization [15].

According to the first and second principle of thermodynamics for quasi-equilibrium

$$de = \delta A + \theta d\eta, \tag{1}$$

where e, A, θ and η are the internal energy density, the work done in an unit volume, the absolute temperature and the entropy density, respectively. In a quadrupole approximation (Brankov and Petrov [15])

$$\delta A = t_{kl} d\varepsilon_{kl} + E_k dP_k + E_{kl} dP_{kl}. \tag{2}$$

The stress tensor, the tensor of the infinitesimal strain, the electric field, the polarization (dipole moment) and the quadrupole moment are denoted by t_{kl}, ε_{kl}, E_k, P_k and P_{kl}. The thermodynamic force E_{kl} conjugated with the quadrupole moment represents the symmetrized gradient of the electric field

$$E_{kl} = (E_{k'l} + E_{l'k})/2. \tag{3}$$

The classical piezoelectric theory represents a dipole approximation without accounting the quadrupole term in (2).

Let us accept that the external electric field is equal to zero and consequently the dipole and quadrupole moment generated on the basis of the electromechanical coupling are small values. By Legendre transformation

$$\Phi = e - t_{kl}\varepsilon_{kl} - E_k P_k - E_{kl} P_{kl} - \theta\eta, \tag{4}$$

equation (1) could be presented in the form

$$d\Phi = -\varepsilon_{kl} dt_{kl} - P_k dE_k - P_{kl} dE_{kl} - \eta d\theta, \tag{5}$$

where Φ is a function of t_{kl}, E_k, E_{kl} and θ. Since Φ is a thermodynamic potential the following equations are valid:

$$\varepsilon_{kl} = -\frac{\partial\Phi}{\partial t_{kl}}, \quad P_k = -\frac{\partial\Phi}{\partial E_k}, \quad P_{kl} = -\frac{\partial\Phi}{\partial E_{kl}}, \quad \eta = -\frac{\partial\Phi}{\partial\theta}. \tag{6}$$

Let us consider the isothermal case and let us approximate Φ by the quadratic form

$$-\Phi = \frac{1}{2} A_{klmn} t_{kl} t_{mn} + \frac{1}{2} \chi_{kl} E_k E_l + \frac{1}{2} C_{klmn} E_{kl} E_{mn} + d^{(1)}_{kln} E_k t_{ln} + D_{klmn} t_{kl} E_{mn} + d^{(2)}_{klm} E_k E_{lm}. \tag{7}$$

The material tensors in (7) satisfy the conditions of symmetry

$$A_{klmn} = A_{mnkl} = A_{lkmn}, \quad \chi_{kl} = \chi_{lk},$$
$$C_{klmn} = C_{mnkl} = C_{lkmn}, \quad d^{(1)}_{klm} = d^{(1)}_{kml},$$
$$D_{klmn} = D_{mnkl} = D_{lkmn}, \quad d^{(2)}_{klm} = d^{(2)}_{kml}.$$

The constitutive equations following from (6) and (7) are:

$$\begin{aligned} \varepsilon_{kl} &= A_{klmn} t_{mn} + d^{(1)}_{nkl} E_n + D_{klmn} E_{mn}, \\ P_k &= d^{(1)}_{kmn} t_{mn} + \chi_{kl} E_l + d^{(2)}_{kmn} E_{mn}, \\ P_{kl} &= D_{klmn} t_{mn} + d^{(2)}_{nkl} E_n + C_{klmn} E_{mn}. \end{aligned} \tag{8}$$

The investigations of Fukada and Yasuda [1, 17], Lang at 1969 and others show that the bone tissue possesses polar hexagonal structure — C_6. In a coordinate system where the axis coincides with the symmetry axis all tensors of 2nd and 3rd rank are presented by the matrices:

$$\{A_{kl}\} \equiv \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & A_{11} & 0 \\ 0 & 0 & A_{33} \end{pmatrix}, \tag{9}$$

$$\{A_{kln}\} \equiv \begin{pmatrix} 0 & 0 & 0 & A_{123} & A_{113} & 0 \\ 0 & 0 & 0 & A_{113} & -A_{123} & 0 \\ A_{311} & A_{311} & A_{333} & 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

Gradient Approximation at Pure Bending

Let us consider a thin plate with lengths of the basic edges a, b and h, which are colinear with the axes X_1, X_2 and X_3, respectively. We assume $h \ll a, b$. In the case of a pure bending around X_1 all components of the stress tensor are equal to zero except t_{22}. From the geometry of the body, in the case of an open electric chain we have [13]

$$D_k = E_k + 4\pi P_k = 0, \quad E_{k,1} = E_{k,2} = 0. \tag{11}$$

By means of $(8)_2$, (9), (10) and (11) we obtain

$$aE_{3,13} + E_3 = dt_{22}, \tag{12}$$

where $a = \frac{4\pi d^{(2)}_{333}}{4\pi\chi_{33}+1}$, $d = \frac{-4\pi d^{(1)}_{311}}{4\pi\chi_{33}+1}$.

After integration it is obtained from equation (12)

$$E_3(z) = E_3(0)e^{-z/a} + \frac{d}{a}\int_0^z e^{-(z-z')/a} t_{22}(z')dz'. \tag{13}$$

The constant a is connected with the quadrupole effect. If it is accepted that it tends to zero, then from (12) and (13) we obtain the classical case

$$E_3 = dt_{22}. \tag{14}$$

Since a is a very small magnitude, the exponential function in equation (13) has value different from zero only in the neighbourhood of z. Therefore it is expedient to use the presentation

$$\frac{1}{a} e^{-z/a} = \mathscr{L}^{-1}\left(\frac{1}{1+as}\right) = \mathscr{L}^{-1}\left[\sum_{k=0}^{\infty}(-a)^k s^k\right] = \sum_{k=0}^{\infty}(-a)^k \frac{d^k}{dz^k}\delta_+(z), \tag{15}$$

where $\mathscr{L}^{-1}$ is a Laplace reverse transformation and $\delta_+(z)$ is asymmetric Dirac delta function [18].

The gradient theory is obtained from (13) accounting only the first two terms in the series (15):

$$E_3(z) = dt_{22} + adt_{22,3}. \tag{16}$$

It is evident that the participation of the stress gradient in equation (16), empirically proposed by Fukada [19] and by Williams and Breger [13] for bending experiments, represents the stress gradient approximation of the quadrupole piezoelectric theory.

From (16) it follows that the potential difference appearing between the opposite surfaces of a thin bone plate at pure bending is

$$\varphi = -adht_{22,3}. \tag{17}$$

From equation (17) follows the well-known but unexplainable by the classical theory result, namely that the change in the sign of the stress gradient causes a change in the sign of the induced electric potential.

Conclusion

The quadrupole model developed here gives the opportunity of studying the electromechanical interaction in a dry bone.

It is shown that the results following from the empiric model proposed by Williams and Breger [13] represent stress gradient approximation of the quadrupole piezoelectric theory.

The theory predicts generation of electric potentials in the case of pure bending of bone plates. When the sign of the stress gradient changes, the sign of the potential changes too.

This model is offered with the knowledge that a similar theo-retical approach, which takes into account second-order effects not of quadrupoles but intrinsic polarization of the material, also predicts a stress-gradient effect.

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