JOURNAL OF THEORETICAL AND APPLIED MECHANICS,
Sofia, 2000, vol. 30, No. 4, pp. 84-92.

(Received 10 January 2000)

ESTIMATION OF THE HYDRAULIC CANALICULAR RADIUS IN CORTICAL BONE

N. PETROV

Institute of Mechanics, Bulgarian Academy of Sciences,

Acad. G. Bonchev Str., Bl. 4, 1113 Sofia, Bulgaria,

e-mail: petrov333@gmail.com

ABSTRACT. The fact that osteocites are not more than 0.1 to 0.2 mm from capillary strongly suggests for canalicular via by which osteocites are nourished and rid themselves of waste products. As possible mechanisms appear to be diffusion and stress generated fluid flow. However it is not clear, what are the efficiency limits of these two ways to transport the species. Because of the presence of cell processes within the canaliculus, it is unknown, what part of the canalicular cross section is occupied and what is free for fluid flow or spices diffusion. The objective of the present study is to answer the question: what is the order of the hydraulic canalicular radius? In the present study its value is estimated about 19 nm. In the present study for first time, in the literature, the hydraulic radius of the canaliculus is directly estimated by processing data of relaxation type experiments

NOMENCLATURE

V, V_o - moment and the reference, load free lacunar volume;

P, P_o - moment and initial lacunar fluid pressure;

R_{eff} - hydraulic (effective) radius of canaliculus;

L - canaliculus length (center-to-center distance between two lacunae);

n - number of canaliculus connecting two lacunae;

M - number of lacunae in one lacunar-canalicular set (Fig. 1);

S - uniaxial stress value;

μ_1 - shear modulus of bone matrix;

K_1 - bulk modulus of bone matrix;

K_w - bulk modulus of fluid;

K_{eff} - effective bulk modulus;

η - viscosity of the fluid;

$\hat{\tau}$ - characteristic lacunar-canalicular system time.

1. Introduction

The fluid flow in bone, from point of view of nutrients supply, has been studied by Jendruco [1], Kelly [2], Dillaman [3], Montgomery et al. [4], Dillaman et al. [5], McCarthy and Yang [6], Keanini et al. [7]. A simple mathematical model for fluid flow in an osteon, consisting of a single lacuna and canaliculus, has been offered by Pollack, Petrov et al. [8]. Many of the previous published observations of stress generated potentials in osteons were explained for first time. Kufahl and Saha [9] extended this mathematical model on the base of the anatomical model, offered by Piekarsky and Munro [10], consisting of canaliculi and lacunae bounded in series as in Fig. 1.

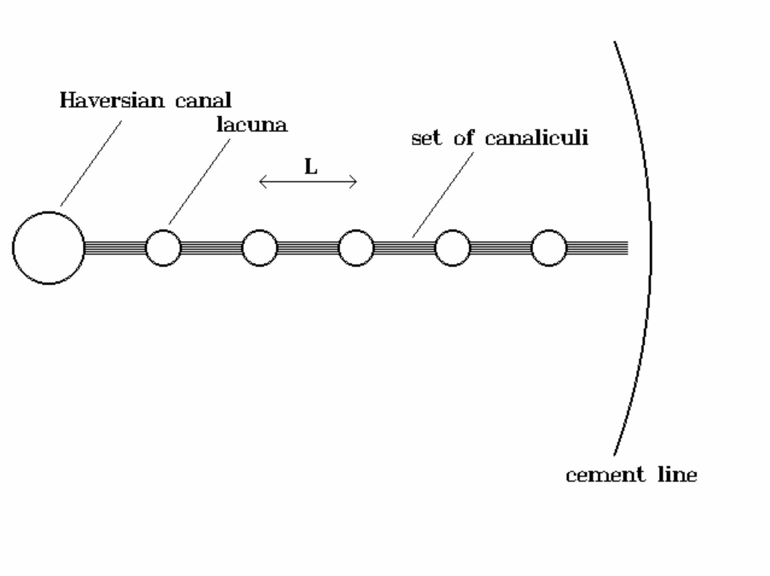

Fig. 1. Haversian canal-lacunae- canaliculi network model of Murno and Piekarski [10]

The question of the effect of stress concentration around the Haversian canal on the symmetry of the fluid flow and streaming potentials has been addressed by Petrov et al. [11] using finite element model to solve Biot [12] equations from the theory of consolidation. Later the model has been generalized by Zeng et al. [13]. The case of fluid flow exchange between the lacunar- canalicular system and microporosity has been studied by Mak et al. [14]. Opposite Weinbaum et al. [15] entirely reject the idea for such a fluid exchange.

The objective of the present study is to give an answer the question: What is the order of the size of the hydraulic canalicular radius? As the canaliculus is plying role of nutrients and waste products transport via, the physiological importance of this problem is without doubt.

2. Physical and mathematical model

Because of the complex microanatomic structure of the osteon, it is reasonable, to make the following simplifying assumptions, as in papers of Pollack et al. [8], Petrov et al. [11], Kufahl and Saha [9], Zeng et al. [13], Weinbaum et al. [15], Wang et al. [16]:

1. Bone matrix is isotropic and elastic;

2. Pores are spherical, fluid filled inclusions;

3. The fluid pressure in the Haversian canal is arterial and is taken to be zero;

4. Dilatation of the pores can be computed as a function of the uniaxial stress;

5. The fluid fluxes at the cement line of osteon are taken to be equal to zero. We have in mind that the fluid pressure has a local extremum at the boundary between two osteons, because of symmetry.

6. All distances between lacunae in series and all canalicular radii are equal.

7. The fluid exchange between lacunar- canalicular system and microporosity is neglected.

Taking into account the above simplifications, we present the fluid flow in lacunar-canalicular system, in case of instantaneous loading, with Kufahl and Saha model [9]:

$$(1) \qquad \frac{d}{dt}\{P\} + \frac{1}{\hat{\tau}}[S]\{P\} = 0,$$

where $\{P\}$ is a vector with components lacunar fluid pressures P^N, $N = 1,2,...,M$ (M is the number of the lacunae in one lacunar-canalicular set),

(2) $$[S] \equiv \begin{pmatrix} 2 & -1 & 0 & 0 & . & . & . & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & . & . & . & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & . & . & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & -1 & 1 \end{pmatrix},$$

is $M \times M$ matrix. The characteristic time $\hat{\tau}$ is function of the microanatomical parameters and physical moduli

(3) $$\hat{\tau} = \frac{8\eta\, L\, V_0}{n\pi R_{eff}^4} \frac{K_w - K_{eff}}{K_w K_{eff}},$$

where V_0 is the reference, load free volume of the lacuna, L is the canaliculus length (center-to-center distance between two lacunae), R_{eff} is the effective (hydraulic) canalicular radius, η is the viscosity of the fluid, K_w is bulk modulus of the bone fluid and K_{eff} is effective bulk modulus defined by the following non steady relation between the lacunar volume V and lacunar pressure P:

(4) $$\frac{dV}{dt} = -\frac{V_0}{K_{eff}} \frac{dP}{dt} \quad , \quad t > 0 .$$

Because of the presence of processes within canaliculus and consequently unknown part of the canalicular cross section is block for fluid flow, opposite to Kufahl and Saha [9], we use in equation (3) the effective radius R_{eff} instead of the anatomical radius of matrix canaliculus interface.

3. Determination of effective bulk modulus

Kufahl and Saha [9] interpreted K_{eff} as an experimentally determined constant. In fact, however we have no available experimental data or experimental method of its determining.

In the present paper an analytic method of explicit determining K_{eff}, for the case of instantaneous loading, is offered. This leads to reduction of the number of the unknown constants and gives possibility of estimating the effective canalicular radius on the base of processing the data from relaxation type experiments. Next the method is represented.

Because bone is consider as elastic, the step load results in an instantaneous dilatation of a fluid filled lacunae (Hashin, [17]):

$$\text{(5)} \qquad \frac{V(0+)-V_0}{V_0}=\frac{4\mu_1+3K_1}{4\mu_1+3K_w}\frac{S}{3K_1} \qquad ,$$

where $V(0+)-V_0$, K_1, μ_1, S are the instantaneous dilatation of the lacuna, shear and bulk moduli of the bone matrix, and the macroscopic uniaxial stress respectively.

The instantaneous pressure P_0 in the lacuna, resulting from dilatation, is

$$\text{(6)} \qquad P_0=-K_w\frac{V(0+)-V_0}{V_0}=-\frac{K_w}{3K_1}\frac{4\mu_1+3K_1}{4\mu_1+3K_w}S\,.$$

Following Pollack, Petrov et al. [8] for the volume balance of lacuna we have:

(7) $$V_0\left[\frac{K_w - K_{eff}}{K_w K_{eff}}\right]\frac{dP}{dt} = J \quad , \quad t > 0 \, ,$$

where J is the sum of all fluxes drained from the inclusion by the system of channels. Integrating the two sides of equation (7) over all time and taking into account that at $t \to \infty$, in the case of instantaneous loading, the dilatation reaches steady state and the pressure of the fluid within the inclusion goes to zero, therefor we obtain

(8) $$-V_0\left[\frac{K_w - K_{eff}}{K_w K_{eff}}\right]P_0 = \int_0^\infty J\,dt = V(\infty) - V_0 \, .$$

Because the lacunae are open for drainage into Haversian canal, their steady state dilatation is characterized with zero fluid pressure and mimic empty pores. This case can be easily obtained from (5), formally substituting $K_w = 0$,

(9) $$\frac{V(\infty) - V_0}{V_0} = \frac{4\mu_1 + 3K_1}{4\mu_1}\frac{S}{3K_1} \, .$$

Substituting equation (6) and (9) into (8) we obtain the following explicit expression of the effective bulk modulus K_{eff}, as function of the fluid bulk modulus K_w and shear modulus of bone matrix μ_1

(10) $$K_{eff} = \frac{4\mu_1 K_w}{8\mu_1 + 3K_w} \, .$$

Inserting (10) into (3) for the characteristic time $\hat{\tau}$ we obtain

$$\hat{\tau} = \frac{2\eta L V_0}{n\pi R_{eff}^4} \frac{4\mu_1 + 3K_w}{\mu_1 K_w} . \tag{11}$$

4. Estimation of the effective canalicular radius

Solving equation (11) with respect to the effective canalicular radius we obtain

$$R_{eff} = \sqrt[4]{\frac{2\eta L V_0}{n\pi\hat{\tau}} \frac{4\mu_1 + 3K_w}{\mu_1 K_w}} . \tag{12}$$

For identification of $\hat{\tau}$, two mean experimental, relaxation type curves were used, first for set of samples invitro and second for set of samples invivo (Otter et al. [18]). The evaluation of the parameter $\hat{\tau}$ was realized with inverse algorithm, based on least-square method.

For a priori selected experimental points $P_l^{\exp r}, l = 1,2,...,m$ and the respective values calculated by the theoretical model $P_l^{calc}(\hat{\tau})$, $l = 1,2,..,m$, the quadratic deviation is defined as

$$R^{dev} = \sum_{l=1}^{m} \left(\frac{P_l^{\exp r}}{P_0^{\exp r}} - \frac{P_l^{calc}}{P_0^{calc}} \right)^2 , \tag{13}$$

where m is the number of selected experimental points, and $P_0^{\exp r}$ and P_0^{calc} are the initial values of the experimental and theoretically calculated pressures, respectively.

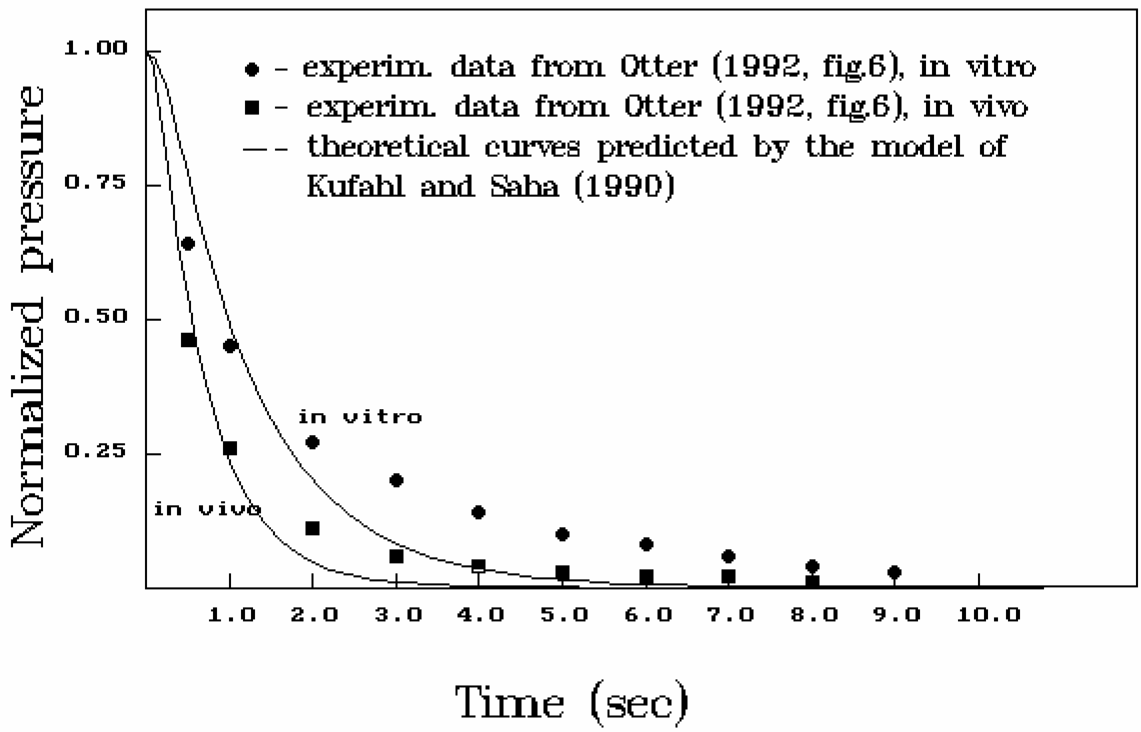

Fig. 2. Normalized experimental data of pressure response at step loading and least square approximations with the model of Kufahl and Saha

The value of $\hat{\tau}$ is determined by minimization of R^{dev} within an apriori, accepted interval. The theoretical values of the pressure in (13) were obtained by numeric integration of equation (1) with Crank-Nickolson differential scheme. The experimental values and their approximation with the model of Kufahl and Saha are represented in Fig. 2.

To determine R_{eff}, beside $\hat{\tau}$ we need of the values of the microanatomical parameters and physical moduli, taking part in formulas (12). For this aim we use reasonable, model anatomical parameters and material moduli as shown in Table 1. The sources for this data are Kelly [2], Moris et al. [19], Pollack et al. [8], Atkinson and Hallsworth [20]. The values of the identified with the above minimization procedure effective canalicular radii R_{eff}, characteristic times $\hat{\tau}$ and the corresponding quadratic deviations R^{dev} are represented in Table 2.

Table 2. Model parameters, identified with the least square method and the respective minimal deviation

Data from	$\hat{\tau}$(sec)	R^{eff} (nm)	$R^{dev}_{\min}$
Otter et al., (1992), invitro	0.09	17.5	0.072
Otter et al., (1992) invivo	0.05	21.4	0.018

5. Discussion and conclusions

A problem arises from the fact that the theoretical model is formulated for single osteon, but for the parameter identification the experimental data for samples, composed of many osteons are used. The reason is the absence of experimental data in the literature, measured on the level of single osteon, (in case of instantaneous loading), but there are experiments on stress generated streaming potentials with whole samples (collection of osteons). The admissibility of such use of experimental data comes from the similarity of the relaxation process in whole samples and in single osteons, if the dominant part of the fluid drainage into Haversian Canals. The experimental and theoretical studies of Otters et al. [21] and MacGinitie et al. [22] support this statement. They have measured experimentally streaming potentials and obtained that the relaxation times do not depend on sample thickness. The conclusion statement is that "there is evidence to indicate that fluid does not flow to the bone surface, but rather flows into Haversian Canals". Cusp-like funnel shape osteonial streaming potentials were experimentally measured by Innoncone et al. [23] at cyclic uniform loading of bone samples. The obtained extremes at the cement lines could be considered as indication that there is not fluid exchange between osteons. Recently, this supposition was theoretically demonstrated by Wang et al. [16] at cyclic non-uniform loading of model sample composed of 6 osteons. It is following from the above studies that the relaxation process is realized dominantly on osteonial level and consequently the relaxation process in whole samples and in single osteons should be analogous. It gives possibility the identification of $\hat{\tau}$ to be realized on the base of processing the experimental data obtained for bone samples, composed of many osteons.

The results, obtained by identification procedure (Table 2) are 17.5 nm for invitro and 21.4 nm for invivo samples. So we may do the statement that the effective canalicular radius about 19 nm. Because of the 4th degree of the root, in equation (12), the predicted values for the effective radius are not sensitive with respect to the values of the anatomical parameters and physical moduli. For example, if the combination of the anatomical parameters and elastic moduli - $\frac{\eta L V_0}{n}\frac{4\mu_1 + 3K_w}{\mu_1 K_w}$ in (12), is varied in limits of ± 500 % the effective radius is varied in the relatively narrow interval: $R_{eff} \in (13.1\ ,\ 29.7)$ nm, e.g. remain at the same order of size.

In the present study for first time, in the literature, the hydraulic radius of the canaliculus is directly estimated by processing data from relaxation type experiments.

ACKNOWLEDGMENTS: The author of the present paper has pleasant obligation to thank Professor Solomon Pollack for his valuable suggestions and supervision of the study, and Professor Stephen Cowin for the copies of papers he has leaven.

REFERENCES

[1] JENDRUCKO, R. J., W. A. HYMAN, P. H. NEWELL, B. K. CHAKRABORTY. Theoretical Evidence for the Generation of High Pressure in Bone Cells. *J. Biomechanics*, **9** (1976), 87-91.

[2] KELLY, P. J. Pathways of Transport in Bone, Handbook of Physiology, The Cardiovascular System, Sect. **2**, **3** (Ed. J. T. Shepherd, F. M. About), American Physiology Society, Bathesda, MD, 1983.

[3] DILLAMAN, R. M. Movement of Ferritin in the two-day Old Chick Femur. *Anat. Res*., **209** (1984), 445-453.

[4] MONTGOMERY, R. J., B. D. SUTKER, J. T. BRONK, S. R. SMITH, P. J. KELLY. Interstitial Fluid Flow in Cortical Bone. *Microvasc. Res*., **35** (1988), 295-307.

[5] DILLAMAN, R. M., R. D. ROER, D. M. GAY. Fluid Movement in Bone: Theoretical and Empirical. *J. Biomechanics*, **24** (1991), 163-177.

[6] MCCARTHY, I. D., L.YANG. A Distributed Model of Exchange Processes Within the Osteon. *J. Biomechanics*, **25** (1992), 441-450.

[7] KEANINI, R. G., R. D. ROER, R. M. DILLAMAN. A Theoretical Model of Circulatory Interstitial Fluid Flow and Species Transport within Porous Cortical Bone. *J. Biomechanics*, **28** (1995), 901-914.

[8] POLLACK, S. R., N. PETROV, R. SALZSTEIN, G. BRANKOV, R. BLAGOEVA. An Anatomical Model for Streaming Potentials in Osteons. *J. Biomechanics*, **17** (1984), 627-636.

[9] KUFAHL, R. H., S. SAHA. A Theoretical Model for Stress - Generated Fluid Flow in the Canaliculi-Lacunae Network in Bone Tissue. *J. Biomechanics*, **23** (1990), 171-180.

[10] PIEKARSKI K., M. MUNRO. Transport Mechanism Operating Between Blood Supply and Osteocytes in Long Bones. *Nature*, **269** (1977), 5623, 80-82.

[11] PETROV, N., S. R. POLLACK, R. BLAGOEVA. A Discrete Model for Streaming Potentials in Single Osteon. *J. Biomechanics*, **22** (1989), 517- 521.

[12] BIOT, M. A. General Theory of Tree-Dimensional Consolidation. *J. Appl. Phys*., **12** (1941), 155-164.

[13] ZENG, Y., S. C. COWIN, S. WEINBAUM. A Fiber Matrix Model for Fluid Flow and Streaming Potentials in the Canaliculi of an Osteon. *Annals of Biomedical Engineering*, **22** (1994), 280-292.

[14] MAK, A. F. T., D. T. HUANG, J. D. ZHANG, T. PIN. Deformation-Induced Hierarchical Flows and Drag Forces in Bone Canaliculi and Matrix Microporosity. *J. Biomechanics*, **30** (1997), 11-18.

[15] WEINBAUM, S., S.C. COWIN, Y. ZENG. A Model for the Excitation of Osteocytes by Mechanical Loading - Induced Bone Fluid Shear Stresses. *J. Biomechanics*, **27** (1994), 339-360.

[16] WANG, L., S. P. FRITTON, S. C. COWIN, S. WEINBAUM. Fluid Pressure Relaxation Depends upon Osteonal Microstructure: Modelling an Oscillatory Bending Experiment. *J. Biomechanics*, **32** (1999), 663-672.

[17] HASHIN, Z. The Inelastic Inclusion Problem. *Int. J. Engng Sci*., **7** (1969), 11-36.

[18] OTTER, W. M., V. R. PALMIERI, D. D.WU, K. G. SEIZ, L. A. MACGINITIE, G. V. B. COCHRAN. A Comparative Analysis of Streaming Potentials in Vivo and Invitro. *J. Orthop. Res.*, **10** (1992), 710-719.

[19] MORRIS, M. A., J. A. LOPEZ-CURTO, S. P. F. HUGHES, K. N. AN., J. B. BASSINGTHWAIGHTE, P. J. KELLY. Fluid Species in Canine Bone and Marrow. *Microvasc. Res.*, **23** (1982), 188-200.

[20] ATKINSON, P. J., A. S. HALLSWORTH. The Spatial Structure of bone. In: Progress in Anatomy, **2**, (Ed. R. J. Harrison, V. Navaratnam), Cambridge University Press, Cambridge, 1982.

[21] OTTER, M. W., D. D. WU, W. A. BIEBER, K. G. SEIZ, G. V. B. COCHRAN. Alternation of Streaming Potentials in Intact Canine Tibiae by Vascular Perfusion with Specific Ionic Solutions: a Pilot Study. *Electro-and-Magnetobiology*, **12** (1993), 85-98.

[22] MACGINITIE, L. A., K. G. SEIZ, M. W. OTTER, G. V. B. COCHRAN. Streaming Potential Measurements at Low Concentrations Reflect Bone Microstructure. *J. Biomechanics*, **27** (1994), 969-978.

[23] IANNACONE, W., E. KOROSTOFF, S. R. POLLACK. Microelectrode Studies of Stress Generated Potentials Obtained From Uniform and Nonuniform Compression of Human Bone. *J. Biomed. Mat. Res*., **13** (1979), 753-763.

[24] PIENKOWSKI, D., S. R. POLLACK. The Origin of Stress Generated Potentials in Fluid Saturated Bone*. J. Orthop. Res*., **1** (1983), 30-41.