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On the Electromechanical Interaction in Physiologic Wet Bones

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The electro-mechanical interaction in human and animal bones under physiologic moisture conditions is a subject of intensive investigations. A theoretical model of this phenomenon is not offered up to now, in spite of the existing numerous experimental data. Some scientists share the opinion that "at the present time mathematical formulation based on a theoretical structure is not possible" [1].

In order to explain the electro-mechanical behaviour of physiologic wet bones the author of this paper offers a thermodynamic model where the bone is considered to be electrically conductive, chemically reactive and electrically polarized continuum [2].

In the present paper some of the results following from the above mentioned theory are investigated and compared with a part of the existing experimental data.

I. Mathematical Model

Numerous investigations show that dry bones, additionally moistured in a physiologic solution react upon a mechanical load with electric signal analogical to the behaviour of the bones "in vivo" [3]. Since the chemical processes do not play any significant role in these cases, they will be neglected in the present considerations. For the sake of further simplification the thermomechanical interaction will be neglected too. It will be admitted also that the examined body is one-dimensional. The basic equations, representing the mathematical model of the examined medium under the above assumptions are:

$$(1.1) \qquad \varrho \frac{\partial^2 u}{\partial t^2} = \frac{\partial T}{\partial x}, \quad 0<x<L, \quad 0<t<\infty,$$

$$(1.2) \qquad \frac{\partial}{\partial t} q + \frac{\partial j}{\partial x} = 0,$$

$$(1.3) \qquad \frac{\partial}{\partial x} \mathcal{D} = 4\pi q,$$

$$(1.4) \qquad \varepsilon = aT + dE,$$

$$(1.5) \qquad \mathcal{D} = dT + \text{э}E,$$

$$(1.6) \qquad j^{(l)} = \sigma^{(l)} E - \varkappa^{(l)} \frac{\partial q^{(l)}}{\partial x},$$

$$(1.7) \qquad q = \sum_{l=1}^{n} q^{(l)},$$

$$(1.8) \qquad j = \sum_{l=1}^{n} j^{(l)},$$

$$(1.9) \qquad \varepsilon = \frac{\partial u}{\partial x},$$

$$(1.10) \qquad E = -\frac{\partial \varphi}{\partial x},$$

$$\varkappa^{(l)},\ \sigma^{(l)},\ a,\ \text{э} > 0,$$

where u, T and ε are the displacement, the mechanical stress and the deformation, respectively; $q^{(l)}$ and $j^{(l)}$ are the electric charge density and the conduction current of the l^{th} kind carriers; q and j are the total density of charge and the total conduction current; E, $\mathcal{D}$ and φ are the electric force, the electric displacement and the electric potential, correspondingly; a, d, э, $\sigma^{(l)}$ and $\varkappa^{(l)}$ are the elastic modulus, the piezoelectric constant, the dielectric permeability, the electric conductivity and the diffusive mobility of the l^{th} kind charge carriers.

For the sake of further simplification of the mathematical model it will be assumed that the considered body is subjected to a quasistatic mechanical load and that all charge carriers are characterized by the same diffusive mobility. Under the above assumptions the basic equations, describing the behaviour of the considered medium are:

$$(1.11) \qquad \frac{\partial}{\partial x} T = 0,$$

$$(1.12) \qquad \frac{\partial}{\partial t} q + \frac{\partial}{\partial x} j = 0,$$

$$(1.13) \qquad \frac{\partial}{\partial x} \mathcal{D} = 4\pi q,$$

$$(1.14) \qquad \varepsilon = aT + dE,$$

$$(1.15) \qquad \mathcal{D} = dT + \text{э}E,$$

$$(1.16) \qquad j = \sigma E - \varkappa \frac{\partial q}{\partial x},$$

(1.17) $$E=-\frac{\partial\varphi}{\partial x},$$

where

$$\sigma=\sum_{l=1}^{n}\sigma^{(l)},\quad \varkappa=\varkappa^{(l)},\quad l=1, 2, \ldots, n.$$

It will be accepted that there is no external electric field acting on the body and the charge carriers do not go out of the sample. Under these assumptions the boundary conditions for $\mathcal{D}$ and j are:

(1.18) $$\mathcal{D}=j=0 \quad \text{for} \quad x=0,\ L,\quad 0<t<\infty.$$

At the initial moment the sample is in its reference state in which the local condition of electroneutrality $q=0$ is satisfied everywhere.

From equations (1.11) — (1.17), from the boundary conditions of $\mathcal{D}$ and j and the initial condition of q the following boundary problem is obtained:

(1.19) $$\frac{\partial}{\partial t}q+\frac{4\pi\sigma}{\epsilon}q=\varkappa\frac{\partial^2 q}{\partial x^2},\quad 0<x<L,\quad 0<t<\infty,$$

(1.20) $$\frac{\partial}{\partial x}q|_{x=0}=\frac{\partial}{\partial x}q|_{x=L}=-\frac{d\sigma}{\epsilon\varkappa}T,\quad 0<t<\infty,$$

(1.21) $$q(x, 0)=0,\quad 0<x<L.$$

Let us consider the case when the mechanical load applied to the sample is a rectangular impulse:

(1.22) $$T=\begin{cases} 0, & \text{at } t<0,\\ \text{const.}, & \text{at } 0\leq t\leq t_0,\\ 0, & \text{at } t>t_0.\end{cases}$$

After calculations, the solution of the boundary problem (1.19 — 1.21) in the time interval $(0, t]$ is found to be in the form:

(1.23) $$q(x, t)=\bar{q}(x)+v(x, t),$$

where

(1.24) $$\bar{q}(x)=\frac{d}{2}\sqrt{\frac{\sigma}{\pi\epsilon\varkappa}}\frac{T}{\left(e^{-2\sqrt{\frac{\pi\sigma}{\epsilon\varkappa}}}-e^{2\sqrt{\frac{\pi\sigma}{\epsilon\varkappa}}L}\right)}\left[\left(1-e^{-2\sqrt{\frac{\pi\sigma}{\epsilon\varkappa}}L}\right)e^{2\sqrt{\frac{\pi\sigma}{\epsilon\varkappa}}x}+\left(1-e^{2\sqrt{\frac{\pi\sigma}{\epsilon\varkappa}}L}\right)e^{-2\sqrt{\frac{\pi\sigma}{\epsilon\varkappa}}x}\right],$$

(1.25) $$v(x, t)=-\frac{a}{2}\cdot e^{-\frac{4\pi\sigma}{\epsilon}t}-\sum_{n=1}^{\infty}a_n e^{-\left(\frac{4\pi\sigma}{\epsilon}+\frac{n^2\pi^2}{L^2}\varkappa\right)t}\cos\frac{n\pi}{L}x,$$

$$a_n=\frac{2}{L}\int_0^L\bar{q}(x)\cos\frac{n\pi x}{L}dx,\quad n=0, 1, 2, \ldots$$

Taking into account the fact that the physiologic wet bones are characterized by a high conductivity σ and a small diffusive mobility $\varkappa$ and neglecting the constants of higher order, it is obtained:

$$(1.26) \qquad \bar{q}(x) = \frac{d}{2}\sqrt{\frac{\sigma}{\pi Э\varkappa}}\, T\left(e^{-2\sqrt{\frac{\pi\sigma}{Э\varkappa}}x} - e^{2\sqrt{\frac{\pi\sigma}{Э\varkappa}}(x-L)}\right),$$

$$(1.27) \qquad v(x,t) = -\frac{4d\sigma T}{LЭ}\sum_{k=0}^{\infty}\frac{e^{-\left[\frac{4\pi\sigma}{Э}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]t}}{\frac{4\pi\sigma}{Э}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}}\cos\frac{(2k+1)\pi x}{L}.$$

The series (1.27) is uniformly converging, since it is majorized by the series

$$I = \frac{4d\sigma L}{Э\pi\varkappa}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^2}$$

the value of which is $\frac{d\sigma L}{2Э\varkappa}$.

From equations (1.13), (1.15), (1.26), (1.27) and the boundary conditions for $\mathcal{D}$, an expression for the electric field E is obtained:

$$(1.28) \qquad E = -\frac{d}{Э}T\left(e^{-2\sqrt{\frac{\pi\sigma}{Э\varkappa}}x} + e^{2\sqrt{\frac{\pi\sigma}{Э\varkappa}}(x-L)}\right)$$

$$-\frac{16d\sigma T}{Э^2}\sum_{k=0}^{\infty}\frac{e^{-\left[\frac{4\pi\sigma}{Э}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]t}}{(2k+1)\left[\frac{4\pi\sigma}{Э}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]}\sin\frac{(2k+1)\pi x}{L}.$$

Integrating equation (1.28) in the interval $(0, L)$ an expression of the potential difference is obtained:

$$(1.29) \qquad \varphi = d\sqrt{\frac{\varkappa}{\pi\sigma Э}}\,T + \frac{32d\sigma LT}{Э^2\pi}\sum_{k=0}^{\infty}\frac{e^{-\left[\frac{4\pi\sigma}{Э}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]t}}{(2k+1)^2\left[\frac{4\pi\sigma}{Э}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]}.$$

In the case when the relation between the diffusive mobility $\varkappa$ and the conductivity σ is a negligibly small value, with a boundary transition $\frac{\varkappa}{\sigma}\to 0$, the following expression for the potential is obtained from (1.29)

$$(1.30) \qquad \varphi = \frac{dL}{Э}\,T e^{-\frac{4\pi\sigma}{Э}t}.$$

If the duration of the applied rectangular stress impulse is sufficiently long the process at the moment t_0 is in its steady state. The corresponding expressions for q, F and φ are:

$$(1.31) \qquad q(x) = \frac{d}{2}\sqrt{\frac{\sigma}{\pi Э\varkappa}}\,T\left(e^{-2\sqrt{\frac{\pi\sigma}{Э\varkappa}}x} - e^{2\sqrt{\frac{\pi\sigma}{Э\varkappa}}(x-L)}\right),$$

(1.32) $$E(x)=-\frac{d}{\vartheta}T\left(e^{-2\sqrt{\frac{\pi\sigma}{\vartheta\varkappa}}x}+e^{2\sqrt{\frac{\pi\sigma}{\vartheta\varkappa}}(x-L)}\right),$$

(1.33) $$\varphi=d\sqrt{\frac{\varkappa}{\pi\sigma\vartheta}}\,T.$$

The distribution of the electric charges $q(x, t)$ after the moment t_0 is determined by the boundary problem:

(1.34) $$\frac{\partial}{\partial t}q+\frac{4\pi\sigma}{\vartheta}q=\varkappa\frac{\partial^2 q}{\partial x^2},\quad 0<x<L,\quad 0<t<\infty,$$

(1.35) $$\frac{\partial}{\partial x}q|_{x=0}=\frac{\partial}{\partial x}q|_{x=L}=0,\quad t_0<t<\infty,$$

(1.36) $$q(x,t_0)=\frac{d}{2}\sqrt{\frac{\sigma}{\pi\vartheta\varkappa}}\,T\left(e^{-2\sqrt{\frac{\pi\sigma}{\vartheta\varkappa}}x}-e^{2\sqrt{\frac{\pi\sigma}{\vartheta\varkappa}}(x-L)}\right).$$

The solution of the boundary problem (1.34)—(1.36) is:

(1.37) $$q(x,t)=-\frac{4d\sigma}{L\vartheta}T\sum_{k=0}^{\infty}\frac{e^{-\left[\frac{4\pi\sigma}{\vartheta}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right](t-t_0)}}{\frac{4\pi\sigma}{\vartheta}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}}\cos\frac{(2k+1)\pi x}{L}.$$

The corresponding solutions of the electric field E and the potential φ are:

(1.38) $$E(x,t)=-\frac{16d\sigma}{\vartheta^2}T\sum_{k=0}^{\infty}\frac{e^{-\left[\frac{4\pi\sigma}{\vartheta}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right](t-t_0)}}{(2k+1)\left[\frac{4\pi\sigma}{\vartheta}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]}\sin\frac{(2k+1)\pi x}{L},$$

(1.39) $$\varphi(t)=\frac{32d\sigma L}{\vartheta^2\pi}T\sum_{k=0}^{\infty}\frac{e^{-\left[\frac{4\pi\sigma}{\vartheta}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right](t-t_0)}}{(2k+1)^2\left[\frac{4\pi\sigma}{\vartheta}+\frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]}.$$

In the case when the diffusive mobility $\varkappa$ is a very small value, with the help of a boundary transition, the following expression of the potential φ is obtained from equation (1.39)

(1.40) $$\varphi(t)=\frac{dL}{\vartheta}e^{-\frac{4\pi\sigma}{\vartheta}(t-t_0)}.$$

II. Discussion on the Obtained Results and their Comparison with the Existing Experimental Data

In Fig. 1 it is given the time-dependence of the generated potential φ at different values of the parameter $\frac{\varkappa}{\sigma}$. A characteristic peculiarity of the curves is the appearance of a steady state potential after the first peak. The dependence between the steady state potential and the applied mechanical load is determined by Eq. (1.33).

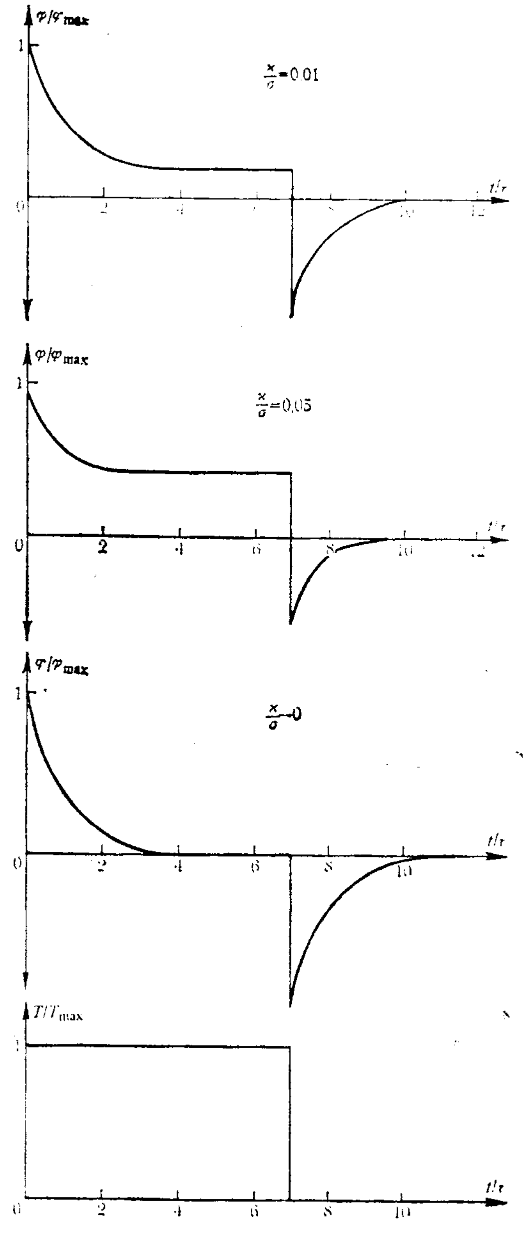

Fig. 1

Such potentials have been registered by numerous scientists [3—6], but they were considered as artefacts. Recently, however, the steady state potentials were systematically studied by Steinberg et al. The results obtained by them confirm the real existence of the steady state potentials as experimental fact. Thus, the obtained experimental relation between the steady state potential and the applied mechanical stress is in conformity with Eq. (1.33).

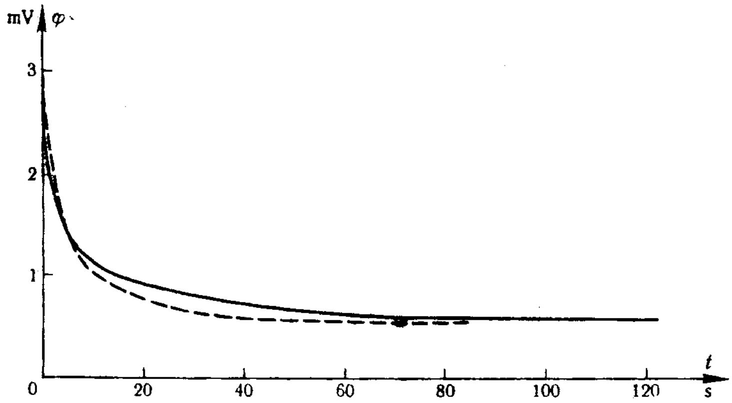

Fig. 2
——— experimental curve [1] ; ········· theoretical curve

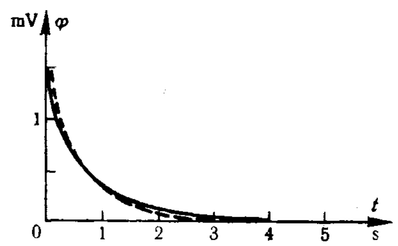

Fig. 3
——— experimental curve [3] ;
········· theoretical curve

It is evident from the presented theory that the steady state potential decreases with the increase of the conductivity σ (Fig. 1). It gives us reason to suppose that this potential is absent at experiments where samples characterized by high conductivity are used [3, 6]. In support of it the following arguments could be presented:

1. The curves obtained by Cochran [3] (Fig. 3) are characterized by a considerably smaller relaxation time than those obtained by Steinberg et al. (Fig. 2) which speaks of a greater number free electric carriers.

2. Cochran [3] also obtains steady state potentials, but rarely. At additional moisturing of the samples the measured potentials reach the zero line. This results could be explained by the offered theory, if it is accepted that the additional moisturing has caused increase in the conductivity of the samples.

The theoretic results are compared with the experimental results, obtained by Steinberg et al. (Fig. 2) and with the experimental ones of Cochran (Fig. 3) in the case of an impulse of a constant load.

A result following from the presented theory is that the motion of the electric carriers introduces relaxation properties in the stress — strain relation. Thus, substituting in Eq. (1.14) the electric field it is obtained from (1.28):

$$\varepsilon = aT - \frac{d^2}{\ni} T \left(e^{-2\sqrt{\frac{\pi\sigma}{\ni\varkappa}}\, x} + e^{2\sqrt{\frac{\pi\sigma}{\ni\varkappa}}\,(x-L)} \right) \tag{2.1}$$

$$- \frac{16 d^2 \sigma}{\ni^2} T \sum_{k=0}^{\infty} \frac{e^{-\left[\frac{4\pi\sigma}{\ni} + \frac{(2k+1)^2\pi^2\varkappa}{L^2}\right] t}}{(2k+1)\left[\frac{4\pi\sigma}{\ni} + \frac{(2k+1)^2\pi^2\varkappa}{L^2}\right]} \sin\frac{(2k+1)\pi x}{L}.$$

Since the second and the third term in (2.1) depend on the quadrate of the piezoelectric constant d, which is a very small value, then their effect will be negligibly small.

III. On the Physical Meaning of the Steady State Potential and its Biologic Significance

The application of a constant mechanical stress on the bone sample causes generation of electric field, which is the reason for the redistribution of the electric charges. At the moment when the diffusion current becomes practically equal to the conduction current

$$\sigma E = \varkappa \frac{\partial q}{\partial x} \tag{3.1}$$

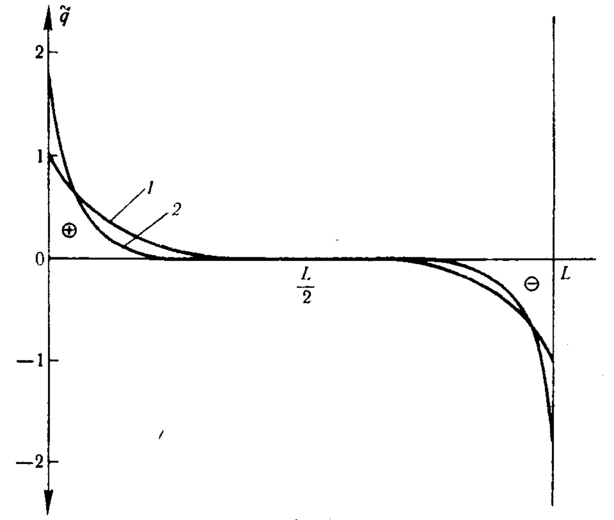

Fig. 4
curve 1 — $\frac{4\pi\sigma}{\ni\varkappa} = 100$; curve 2 — $\frac{4\pi\sigma}{\ni\varkappa} = 400$, $\tilde{q} = \frac{4\pi}{dT} q$

a new stationary distribution of the electrical charges, defined by Eq. (1.31) occurs. In Fig. 4 it is presented the distribution of the electrical charges at different values of $\frac{\varkappa}{\sigma}$. It is evident that near to the boundary surfaces a spacial area with increased concentration of positive or negative electric charges appear. On the other hand, it is a well known fact that the increased concentration of some ions and especially those of Ca^{++} [7—9] influences the metabolic processes in the cells.

In the present investigations the bone sample is considered to be a homogeneous anisotropic body. In fact, however, the bone is morphologically and mechanically non-homogeneous. In many neighbouring microareas the material symmetry and the mechanical properties change rapidly. That is why it would be more correct the bone to be treated as a superposition of micro-areas. In each one of them a non-homogeneous distribution of the electrical charges occurs under the effect of the mechanical stress. The electric distribution of every microarea is analogical to that presented in Fig. 4. Thus, under the action of a mechanical stress a complex electrical picture is formed in the bone, which will repeat to a certain extent the morphological structure. Taking into account the available information about the interrelation between the ionic surrounding and the metabolic processes in the cell, it could be expected that in the electrically charged areas of the bone a positive or negative stimulation of the cell activity will occur.

Since the steady state potentials are directly connected with the non-homogeneous distribution of the electric charges, the author is inclined to believe that they possess definite, biologically meaningful information.

IV. Conclusion

In the present paper a part of the results, following from the offered by the author [2] theoretical model of electro-mechanical interaction in physiologic wet bones are studied. It is shown that:

— the proposed theory is in good coincidence with the experimental results;

— the calculation of the diffusion of the electrical charges leads to:

a) existence of a steady state potential;

b) non-homogeneous distribution of the electrical charges.

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References

1. Steinberg, M. E., G. L. Bunsenkell, J. Black, E. Korostoff. Stress-Induced Potentials in Moist Bone in Vitro. — The Journal of Bone and Joint Surgery, **56-A**, 4, 1974, 704.
2. Petrov, N. Electro-Mechanical Interaction in Physiologic Wet Bone. — Biomechanics, Sofia, 1975, 2.
3. Cochran, G. V. Electro-Mechanical Characteristics of Moist Bone. D. M. Sc. Thesis, Columbia College of Physicians and Surgeons, 1966.
4. Bassett, C. A., R. O. Becker. Generation of Electric Potentials by Bone in Response to Mechanical Stress. — Science, **137**, 1962, 1063.
5. Becker, R. O., C. A. Bassett, C. H. Bachman. Bioelectric Factors Controlling Bone Structure. — In: Bone Biodynamics, Boston, Little Brown, 1964.

6. Cochran, G. V., R. J. Pawluk, C. A. Bassett. Electro-Mechanical Characteristics of Bone under Physiologic Moisture Conditions. — Clinical Orthopaedics and Related Research, **58**, 1968, 251.
7. Whitfield, J., T. Yondale. — Exp. Cell Res., **43**, 1969, 602.
8. Perris, A., J. Whitfield. — Nature, **216**, 1967, 1350.
9. Perris, A., J. Whitfield, R. Rixon. — Radiat. Res., **32**, 1967, 550.

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К вопросу об электромеханических взаимодействиях в физиологически мокрых костях

Н. Петров

(Резюме)

На базе ранее предложенной теоретической модели в настоящей работе решена задача генерирования пьезоэлектрических потенциалов при подаче импульса от постоянного механического напряжения. Результаты сравниваются с частью существующих экспериментальных данных. Дается теоретическое объяснение регистрированным потенциалам. Показано, что с повышением электрической проводимости и понижением диффузионной подвижности эти потенциалы понижаются.