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Electro-mechanical Interaction in Physiologic Wet Bones

Nikola Petrov

Central Laboratory of Biomechanics, Sofia

The generation of electrical charges in human and animal bones under mechanical actions was observed for the first time by Yasuda in 1955 [1]. The classical works of Fukada, Yasuda, Bassett, Becker, Shamos and Lavine [2—6] prove that the observed phenomenon is due to the piezoelectric properties of the bone.

It is considered that the electromechanical interaction in the living tissues has fundamental biological function. Thus, according to the bone remodelling theory, offered by Bassett and Becker [4, 7], the electrical processes, stimulated by the deformation, influence the bone formation. The possibility of applying the obtained results for control of the skeletal architecture remodelling is the reason for the intensive study of these processes [8, 9]. The electromechanical behaviour of physiologically wet bones is of particular interest. [10—12]. The experimental results, obtained in this case, show the existence of high charge conductivity which limits the validity of the basic equations of the classical piezoelectric theory, concerning, as it is well known, dielectric bodies. In 1962 Bassett and Becker made critical remarks on the possibility of describing the electromechanical interaction of biological tissues by the existing classical theory [4]. This problem is considered in [13].

In the present studies it is proposed a thermodynamic model of an electrically polarized body in which the charge conductivity processes and those of the chemical transformations are taken into account.

The studied body is considered as a superposition of electrically polarized bone matrix and n ionic continua. It is supposed that the electroneutrality condition for the bone matrix is satisfied. The electric conduction is realized by the relative ionic continua motion with respect to the basic matrix. The existing interaction between the different constituents of the superimposed body is characterized by mass, electric charge, momentum, energy and entropy exchange. Since the partial mass densities of the ionic continua are small magnitudes, it will be assumed that in every point of the deformed body the mean velocity of the "mixture" is practically equal to the individual velocity of the bone matrix.

I. Kinematics

Each constituent is characterized by an individual equation of motion

(1.1) $$x_k = \chi_k^{(l)}(X_K^{(l)}, t), \quad l=0, 1, 2, \ldots, n.$$

$X_K^{(l)}$ are Cartesian co-ordinates by which we parametrize the undeformed state of the lth constituent of the body. x_k are Cartesian co-ordinates of the material points of all constituents which at a moment t occupy position x in the deformed state of the body.

All values marked by $l=0$ are related to the bone matrix.

The individual velocities of the body constituents are:

(1.2) $$v_k^{(l)} = \frac{\partial}{\partial t} \chi_k^{(l)}(X_K^{(l)}, t) \Big|_{X_K}, \quad l=0, 1, \ldots, n.$$

The relative velocity of the lth ionic continuum with respect to the basic matrix is:

(1.3) $$u_k^{(l)} = v_k^{(l)} - v_k^{(0)}, \quad l=1, 2, \ldots, n.$$

The material derivative referred to the lth continuum is:

(1.4) $$\frac{D^{(l)}}{Dt} = \frac{\partial}{\partial t}\Big|_x + v^{(l)} . \mathrm{grad}.$$

II. Balance Equations

Partial Balance Equations

As it is noted already, each body constituent represents an open system. That is why, in the local balance equations of the partial values, additional terms, accounting the exchange between the components will be involved [**14**].

1. Partial Mass Balance Equation:

(2.1) $$\frac{\partial}{\partial t} \varrho^{(l)} + (\varrho^{(l)} v_k^{(l)})_{,k} = \hat{\varrho}^{(l)}, \quad l=0, 1, 2, \ldots, n,$$

where $\varrho^{(l)}$ and $\hat{\varrho}^{(l)}$ are respectively the partial mass density and the rate of the mass supply of the lth body constituent.

2. Partial Electric Charge Balance Equation:

(2.2) $$\frac{\partial}{\partial t} q^{(l)} + j_{k,k}^{(l)} = \hat{q}^{(l)}, \quad l=1, 2, \ldots, n,$$

where $q^{(l)}$, $j^{(l)}$ and $\hat{q}^{(l)}$ are the partial electric charge density, the current due to the motion of the lth ionic continuum, and the rate of the partial electric charge supply, respectively. The following relations exist between the magnitudes $q^{(l)}$, $j_k^{(l)}$, $\hat{q}^{(l)}$ and $\varrho^{(l)}$, $v^{(l)}$, $\hat{\varrho}^{(l)}$:

$$q^{(l)}=z^{(l)}\mathcal{F}\varrho^{(l)},$$
$$j^{(l)}=z^{(l)}\mathcal{F}v^{(l)}, \tag{2.3}$$
$$\hat{q}^{(l)}=z^{(l)}\mathcal{F}\hat{\varrho}^{(l)},$$

where $z^{(l)}$ is the electrochemical valency of the lth continuum particles, and $\mathcal{F}$ is Faraday constant.

3. Partial Momentum Balance Equation. In the general case the components of a mixture are under the action of volume and surface forces. The balance equation for each constituent is [14]:

$$T^{(l)}_{km,k}+\varrho^{(l)}\left(f^{(l)}_m-\frac{D^{(l)}}{Dt}v^{(l)}_m\right)=\hat{p}^{(l)}_m, \tag{2.4}$$

where $T^{(l)}_{k,m}$, $\varrho^{(l)}f^{(l)}_m$ and $\hat{p}^{(l)}_m$ are the partial stress tensor, the volume force density and the rate of the momentum supply due to the interaction between the components.

In the present consideration the volume forces have electric nature and for the different continua they are:

$$f^{(0)}_m=E_{m,k}\Pi_k,$$
$$f^{(l)}_m=z^{(l)}\mathcal{F}E_m,\quad l=1,2,\dots,n. \tag{2.5}$$

$(2.5)_1$ is the force with which the effective electric field E_m acts per unit mass of a polarized dielectric. Π_k is the polarization per unit mass. $(2.5)_2$ is the force with which the electric field acts per unit mass of an ionic continuum with electrochemical valency $z^{(l)}$.

In the momentum balance equations the influence of the magnetic field is neglected, since it is well known that it affects only high rate processes [15].

Taking into account the physical nature of the ionic continua, it will be accepted that they are not under the action of surface forces, i. e.

$$T^{(l)}_{km}=0\quad \text{at}\quad l=1,2,\dots,n. \tag{2.6}$$

4. Moment of Momentum Partial Balance Equation. In each unit volume of the bone matrix, considered as a polarized dielectric, acts an electric couple [16]

$$\mathcal{L}_k=\varrho^{(0)}e_{kij}\Pi_iE_j, \tag{2.7}$$

where e_{kij} is Levi-Civita tensor. Consequently, the stress tensor $T^{(0)}_{ij}$ is not symmetrical

$$T^{(0)}_{ij}-T^{(0)}_{ji}=\varrho^{(0)}(\Pi_jE_i-E_j\Pi_i). \tag{2.8}$$

5. Partial Energy Balance Equations. The energy of the bone matrix is balanced by the work done by the surface and the electric forces, by the heat supply and by the energy supply due to the interaction with the other constituents

$$\varrho^{(0)}\frac{D}{Dt}\varepsilon^{(0)}=T^{(0)}_{km}v^{(0)}_{m,k}+\varrho^{(0)}E_k\frac{D}{Dt}\Pi_k+h^{(0)}_{k,k}+\hat{\varepsilon}^{(0)} \tag{2.9}$$

$$\left(\frac{D}{Dt}=\frac{D^{(l)}}{Dt}\right).$$

$\varepsilon^{(0)}$ is the partial internal energy density, $\hat{\varepsilon}^{(0)}$ is the rate of the internal energy supply, $h^{(0)}_k$ is the partial heat flux.

The ionic continua are not electrically polarized and they are not under the action of surface forces and consequently the internal energy density $\varepsilon^{(l)}$ is balanced only by the partial heat flux $h^{(l)}_k$ and the rate of the internal energy supply $\hat{\varepsilon}^{(l)}$:

$$\varrho^{(l)}\frac{D^{(l)}}{Dt}\varepsilon^{(l)}=h^{(l)}_{k,k}+\hat{\varepsilon}^{(l)},\quad l=1,2,\ldots,n. \tag{2.10}$$

6. Entropy Unequality. According to the second principle of thermodynamics each constituent satisfies the unequality:

$$\theta\varrho^{(l)}\frac{D^{(l)}}{Dt}\eta^{(l)}-h^{(l)}_{k,k}+\frac{\theta_{1,k}}{\theta}-\theta\hat{\eta}^{(l)}\geq 0,\quad l=0,1,\ldots,n, \tag{2.11}$$

where $\eta^{(l)}$ is the partial entropy density, θ is the thermodynamic temperature of the body, which is accepted to be the same for all constituents, and $\hat{\eta}^{(l)}$ is the rate of entropy supply in the lth continuum, resulting from its interaction with the other constituents.

Total Balance Equations

In the studied medium the principles of conservation could be satisfied not only for the separate constituents, but for the total body as well. In case of electrically neutral mixture, the necessary conditions are as follows [14]:

$$\sum_{l=0}^{n}\hat{\varrho}^{(l)}=0, \tag{2.12}$$

$$\sum_{l=0}^{n}(\hat{p}^{(l)}_k+\hat{\varrho}^{(l)}u^{(l)}_k)=0, \tag{2.13}$$

$$\sum_{l=0}^{n}\left[\hat{\varepsilon}^{(l)}+\hat{p}^{(l)}_k u^{(l)}_k+\hat{\varrho}^{(l)}\left(\varepsilon^{(l)}+\frac{1}{2}u^{(l)}_k u^{(l)}_k\right)\right], \tag{2.14}$$

$$\sum_{l=0}^{n}(\hat{\eta}^{(l)}+\hat{\varrho}^{(l)}\eta^{(l)}). \tag{2.15}$$

The physical meaning of equations (2.12)—(2.15) is that the interaction between the body constituents does not cause change of mass, momentum, energy and entropy of the total body.

Since the body under consideration is electrically charged, the following equation has to be added to equations (2.12)—(2.15):

$$\sum_{l=1}^{n}\mathcal{F}z^{(l)}\hat{\varrho}^{(l)}=0. \tag{2.16}$$

Equation (2.16) expresses the fact that the electric charge of the total body is conserved.

The following relations, describing the total properties of the body are obtained after summing equations (2.1)—(2.11) by l and using equations (2.12)—(2.16):

$$\frac{\partial}{\partial t}\varrho+(\varrho v_k)_{,k}=0, \tag{2.17}$$

$$\frac{\partial}{\partial t}q+j_{k,k}=0, \tag{2.18}$$

$$T_{km,k}+qE_m+\varrho E_{m,k}\Pi_k-\varrho\frac{D}{Dt}v_m=0, \tag{2.19}$$

$$\varrho\frac{D}{Dt}\varepsilon-T_{km}v_{m,k}-\varrho E_m\frac{D}{Dt}\Pi_m-h_{k,k}-E_k\tilde{j}_k=0, \tag{2.20}$$

$$\varrho\frac{D}{Dt}\eta-\left(\frac{h_k}{\Theta}+\sum_{l=1}^{n}\frac{u^{(l)}}{\vartheta}I_k^{(l)}\right)_{,k}\geqq 0, \tag{2.21}$$

where

$$\varrho=\sum_{l=0}^{n}\varrho^{(l)}\approx\varrho^{(0)} \tag{2.22}$$

is the mass density,

$$v=\frac{1}{\varrho}\sum_{l=0}^{n}\varrho^{(l)}v^{(l)}\approx v^{(0)} \tag{2.23}$$

is the mean velocity of the body,

$$q=\sum_{l=1}^{n}q^{(l)}=\sum_{l=1}^{n}z^{(l)}\mathcal{F}\varrho^{(l)} \tag{2.24}$$

is the electric charge density,

$$j_k=\sum_{l=1}^{n}j^{(l)}=\sum_{l=1}^{n}z^{(l)}\varrho^{(l)}v^{(l)} \tag{2.25}$$

is the total electric current,

$$T_{km}=T_{km}^{(0)}-\sum_{l=1}^{n}\varrho^{(l)}u_k^{(l)}u_m^{(l)} \tag{2.26}$$

is the total stress tensor. From (2.26) and (2.8) it follows:

$$T_{km}-T_{mk}=\varrho(E_k\Pi_m-E_m\Pi_k).$$

$$\varepsilon=\frac{1}{\varrho}\sum_{l=0}^{n}\left(\varrho^{(l)}\varepsilon^{(l)}+\frac{1}{2}\varrho^{(l)}u_k^{(l)}u_k^{(l)}\right) \tag{2.27}$$

is the total internal energy density,

$$h_m = \sum_{l=0}^{n} h_m^{(l)} - \sum_{l=1}^{n} \left(\varrho^{(l)} \varepsilon^{(l)} + \frac{1}{2} \varrho^{(l)} u_k^{(l)} u_k^{(l)} \right) u_m^{(l)} \tag{2.28}$$

is the total heat flux,

$$\tilde{j}_k = \sum_{l=1}^{n} q^{(l)} u_k^{(l)} = \sum_{l=1}^{n} z^{(l)} \varrho^{(l)} u_k^{(l)} = \sum_{l=1}^{n} \tilde{j}_k^{(l)} \tag{2.29}$$

is the conductivity current,

$$T_k^{(l)} = \varrho^{(l)} u_k^{(l)} \tag{2.30}$$

is the l th mass flux related to the basic matrix.

With the help of (2.17) and (2.30) the partial mass balance equations obtain the form:

$$\varrho \frac{D}{Dt} c^{(l)} + I_{k,k}^{(l)} = \hat{\varrho}^{(l)}, \tag{2.31}$$

where $c^{(l)} = \varrho^{(l)}/\varrho$ is the concentration of the l th constituent.

$$E_k \tilde{j}_k = \mathcal{F} E_k \sum_{l=1}^{n} z^{(l)} I_k^{(l)} \tag{2.32}$$

is Joule heat supply due to current conductivity.

$$\eta = \frac{1}{\varrho} \sum_{l=0}^{n} \varrho^{(l)} \eta^{(l)} \tag{2.33}$$

is the total entropy density.

$$\mu^{(l)} = \left(\varepsilon^{(l)} + \frac{1}{2} u_k^{(l)} u_k^{(l)} - \theta \eta^{(l)} \right) \tag{2.34}$$

is the chemical potential of the l th body constituent.

III. Electrostatic Equations

As it was noted above, the effect of the magnetic field is considerable only at high rate processes. That is why its influence could be neglected. The basic equations of the electric field expressed in Gaussian unit system have the form:

$$e_{kml} E_{l,m} = 0 \tag{3.1}$$

$$\mathcal{D}_{k,k} = 4\pi q \tag{3.2}$$

where

$$\mathcal{D}_k = E_k + 4\pi \varrho \Pi_k \tag{3.3}$$

is the electrical displacement.

From equation (3. 1) it follows that the electric field admits potential, i. e.

(3.4) $$E_k = -\varphi_{,k}$$

where φ is the potential function.

IV. Constitutive-Equations

The experimental studies carried out by Bassett and Cochran show that the relation between the generated potential, the deformation and the stress in wet bones has initial region in which the corresponding curves are linear [7, 10, 11]. Thus, as a first step the linear constitutive equations will be considered. Furthermore, regardless of the viscoelastic properties of the bones, the elastic theory will be applied and the generalized forces η, T_{km}, E_k, $\mu^{(l)}$ will not depend on the generalized rates. It is assumed that the bone is not under the effect of an external electric field. Consequently, the electric field will be fully generated by the polarization and the nonequilibrium distribution of the electric charges. Since $E_k\Pi_m$ in (2. 8) is nonlinear term it could be neglected and the tensors $T^{(0)}_{km}$ and T_{km} will be considered symmetric.

By means of equations (2. 20) and (2. 31) the entropy unequality could be presented in the form:

(4.1) $$\varrho\left(\theta\frac{D}{Dt}\eta-\frac{D}{Dt}\varepsilon\right)+T_{km}v_{m,k}+\varrho E_m\frac{D}{Dt}\Pi_m+\sum_{l=1}^{n}\varrho\mu^{(l)}\frac{D}{Dt}c^{(l)} -\sum_{l=1}^{n}(\mu^{(l)}+z^{(l)}\mathcal{F}\varphi)_{,k}\quad I_k^{(l)}-\sum_{l=1}^{n}\mu^{(l)}\hat{\varrho}^{(l)}-\theta_{,k}I_k^{\eta}\geqq 0,$$

where

(4.2) $$I_k^{\eta}=\frac{h_k}{\theta}+\sum_{l=1}^{n}\frac{\mu^{(l)}}{\theta}I^{(l)}$$

is the entropy flux.

By means of the values

(4.3) $$\psi=\varepsilon-\theta\eta,$$

(4.4) $$\tilde{\mu}^{(l)}=\mu^{(l)}+z^{(l)}\mathcal{F}\varphi$$

representing the free energy density and the electro-chemical potentials and with the help of unequality (4.1), the second thermodynamic principle obtains the form:

(4.5) $$\varrho\left(\frac{D\psi}{Dt}+\eta\frac{D}{Dt}\theta\right)-T_{km}v_{m,k}-\varrho E_m\frac{D}{Dt}\Pi_m-\sum_{l=1}^{n}\varrho\mu^{(l)}\frac{D}{Dt}c^{(l)} +\sum_{l=1}^{n}\tilde{\mu}^{(l)}_{,k}I_k^{(l)}+\theta_{,k}I_k^{\eta}+\sum_{l=1}^{n}\mu^{(l)}\hat{\varrho}^{(l)}\leqq 0.$$

It is postulated that ψ is a function of the small strain tensor

$$\varepsilon_{ij}=\frac{1}{2}\left[\frac{\partial}{\partial X_i^{(0)}}(x_j-X_j^{(0)})+\frac{\partial}{\partial X_j^{(0)}}(x_i-X_i^{(0)})\right]$$

the temperature deviation θ from its equilibrium value θ_0, the concentration deviations $c^{(l)}$ from their equilibrium values $c_0^{(l)}$ and the polarization Π_m. Considering the dependence of ψ on the above mentioned values in (4.5), the following unequality is obtained:

$$(4.6)\qquad \varrho\left(\frac{\partial\psi}{\partial\theta}+\eta\right)\frac{D}{Dt}\theta+\varrho\left(\frac{\partial\psi}{\partial\varepsilon_{kl}}-\frac{1}{\varrho}T_{kl}\right)\frac{D}{Dt}\varepsilon_{kl}+\varrho\left(\frac{\partial\psi}{\partial\Pi_k}-E_k\right)\frac{D}{Dt}\Pi_k$$
$$+\sum_{l=1}^{n}\varrho\left(\frac{\partial\psi}{\partial c^{(l)}}-\mu^{(l)}\right)\frac{D}{Dt}c^{(l)}+\sum_{l=1}^{n}\hat{\mu}_{,k}^{(l)}I_k^{(l)}+\theta_{,k}I_k^{\eta}+\sum_{l=1}^{n}\mu^{(l)}\hat{\varrho}^{(l)}\leq 0.$$

The free energy density ψ is presented in the form:

$$(4.7)\qquad \psi=\frac{1}{2}A_{ijkl}\varepsilon_{ij}\varepsilon_{kl}+\frac{1}{2}B_{ij}\Pi_i\Pi_j+\frac{1}{2}k(\theta-\theta_0)^2$$
$$+\frac{1}{2}\sum_{l,m=1}^{n}N^{(lm)}(c^{(l)}-c_0^{(l)})(c^{(m)}-c_0^{(m)})+E_{ijk}\varepsilon_{ij}\Pi_k+K_{ij}\varepsilon_{ij}(\theta-\theta_0)$$
$$+\sum_{l=1}^{n}\mathcal{F}_{ij}^{(l)}\varepsilon_{ij}(c^{(l)}-c_0^{(l)})+M_i\Pi_i(\theta-\theta_0)+\sum_{l=1}^{n}G_i^{(l)}\Pi_i(c^{(l)}-c_0^{(l)})$$
$$+\sum_{l=1}^{n}H^{(l)}(\theta-\theta_0)(c^{(l)}-c_0^{(l)}),$$

where

$$A_{ijkl}=A_{klij}=A_{jikl},\quad B_{ij}=B_{ji},\quad E_{ijk}=E_{jik},$$
$$K_{ij}=K_{ji},\quad \mathcal{F}_{ij}^{(l)}=\mathcal{F}_{ji}^{(l)}.$$

Since the terms in brackets in (4.6) do not depend on the generalized rates $\frac{D}{Dt}\theta$, $\frac{D}{Dt}\varepsilon_{km}$, $\frac{D}{Dt}\Pi_k$ and $\frac{D}{Dt}c^{(l)}$, the following relations are obtained from (4.6) and (4.7):

$$(4.8)\qquad -\eta=K_{ij}\varepsilon_{ij}+M_i\Pi_i+\sum_{l=1}^{n}H^{(l)}(c^{(l)}-c_0^{(l)})+K(\theta-\theta_0),$$

$$(4.9)\qquad \frac{1}{\varrho}T_{km}=A_{kmij}\varepsilon_{ij}+E_{kmi}\Pi_i+\sum_{l=1}^{n}\mathcal{F}_{km}^{(l)}(c^{(l)}-c_0^{(l)})+K_{kjm}(\theta-\theta_0),$$

$$(4.10)\qquad E_k=E_{ijk}\varepsilon_{ij}+B_{ki}\Pi_i+\sum_{l=1}^{n}G_k^{(l)}(c^{(l)}-c_0^{(l)})+M_k(\theta-\theta_0),$$

$$(4.11)\qquad \mu^{(l)}=\mathcal{F}_{ij}^{(l)}\varepsilon_{ij}+G_i^{(l)}\Pi_i+\sum_{m=1}^{n}N^{(lm)}(c^{(m)}-c_0^{(m)})+H^{(l)}(\theta-\theta_0),$$

$$(4.12)\qquad \sum_{l=1}^{n} \tilde{\mu}^{(l)}_{,k} I^{(l)}_k + \theta_{,k} I^{\eta}_k + \sum_{l=1}^{n} \mu^{(l)} \hat{\varrho}^{(l)} \leq 0.$$

In unequality (4.12) the values $\tilde{\mu}^{(l)}_{,k}$; $\theta_{,k}$; $\mu^{(l)}$ represent generalized thermodynamic forces, and $I^{(l)}_k$; I^{η}_k; $\hat{\varrho}^{(l)}$ — generalized thermodynamic fluxes. According to Onsager's theory, valid for systems in state near to thermodynamical equilibrium the following relation exists:

$$(4.13)\qquad \begin{pmatrix} I^{(l)} \\ I^{\eta} \\ \hat{\varrho}^{(l)} \end{pmatrix} = \{\mathscr{L}\} \begin{pmatrix} \text{grad}\, \tilde{\mu}^{(l)} \\ \text{grad}\, \theta \\ \mu^{(l)} \end{pmatrix},$$

where $\{\mathscr{L}\} = \{\mathscr{L}\}^T$ is negative definite matrix. Since $I^{(l)}_k$ and I^{η}_k vanish at $\hat{\mu}^{(l)}_{,k} = \theta_{,k} = 0$ the matrix equation (4.13) is separated into the following two independent system equations:

$$(4.14)\qquad \begin{aligned} I^{(l)}_k &= \sum_{s=1}^{n} \lambda^{(ls)}_{km} \tilde{\mu}^{(s)}_{,m} + \lambda^{(ln-1)}_{km} \theta_{,m}, \\ I^{\eta}_k &= \sum_{s=1}^{n} \lambda^{(n+1s)}_{km} \tilde{\mu}^{(s)}_{,m} + \lambda^{(n-1n-1)}_{km} \theta_{,m}, \end{aligned}$$

$$(4.15)\qquad \hat{\varrho}^{(l)} = \sum_{s=1}^{n} \gamma^{(ls)} \mu^{(s)},$$

where

$$\lambda^{(ls)}_{km} = \lambda^{(sl)}_{km} = \lambda^{(ls)}_{mk}, \quad \gamma^{(ls)} = \gamma^{(sl)}$$

are negative definite matrices.

Solving equation (4.10) with respect to Π_k it is obtained that the electric polarization of the medium is determined by the electric field, by the temperature, the deformation and the nonequilibrium concentration of the charge carriers. In the model under consideration the bone matrix and the ionic continua are treated as continua filling the total body. In fact, the ionic continua are isolated in phase from the bone matrix, since the current conductivity is realized mainly in the bone pores, filled with high conductivity physiological liquid. Because of that reason the polarization of the bone matrix should depend on the ionic continua concentration only by the local electric field created by them. This effect, however, is accounted in the effective electric field E_m with the help of equation (3.2). Consequently, Π_m should not be directly dependable on $c^{(l)}$. Grounding on similar considerations based on the phase isolation of the bone matrix and of the charge carriers, the stresses in (4.9) connected with the nonequilibrium concentrations $c^{(l)}$ could be neglected.

The constitutive equations following from the above assumptions and from equations (4.8)—(4.11) are:

$$(4.16)\qquad -\eta = K_{ij}\varepsilon_{ij} + M_i\Pi_i + \sum_{l=1}^{n} H^{(l)}(c^{(l)} - c_0^{(l)}) + K(\theta - \theta_0),$$

$$(4.17)\qquad \frac{1}{\varrho_0} T_{kl} = A_{klij}\varepsilon_{ij} + E_{kli}\Pi_i + K_{kl}(\theta - \theta_0),$$

$$(4.18)\qquad E_k = E_{ijk}\varepsilon_{ij} + B_{ki}\Pi_i + M_k(\theta - \theta_0),$$

$$(4.19)\qquad \mu^{(l)} = \sum_{m=1}^{n} N^{(lm)}(c^{(m)} - c_0^{(m)}) + H^{(l)}(\theta - \theta_0).$$

Since the presented constitutive theory is linear, the mass density ϱ in (4.9) is substituted by the initial density ϱ_0.

By means of (4.19), the transport equations (4.14) are presented in the form:

$$(4.20)\qquad I_k^{(l)} = \sum_{s=1}^{n} \lambda_{km}^{(ls)} \left(\sum_{r=1}^{n} N^{(sr)} c_{,m}^{(r)} - z^{(s)} \mathcal{F} E_m \right) + \left(\lambda^{(ln+1)} + \sum_{s=1}^{n} \lambda_{km}^{(ls)} H^{(s)} \right) \theta_{,m}.$$

In the above equation the electrochemical valency $z^{(l)}$, of all ionic continua are included. From a physical point of view each ionic current depends only on its charge state. This condition is satisfied if:

$$(4.21)\qquad \lambda_{km}^{(ls)} = \lambda_{km}^{(l)} \delta^{ls},$$

where δ^{ls} is Croniker symbol. Using (4.21) in (4.20) it is obtained:

$$(4.22)\qquad I_k^{(l)} = \lambda_{km}^{(l)} N^{(ll)} c_{,m}^{(l)} - \lambda_{km}^{(l)} z^{(l)} E_m + (\lambda_{km}^{(ln+1)} + \lambda_{km}^{(ll)} H^{(l)}) \theta_{,m}.$$

The proposed theory includes the case when some continua are electrically neutral, i. e. $z^{(l)} = 0$. In this case the following transport equations are valid:

$$(4.23)\qquad I_k^{(l)} = \lambda_{km}^{(l)} N^{(ll)} c_{,m}^{(l)} + (\lambda_{km}^{(ln+1)} + \lambda_{km}^{(ll)} H^{(l)}) \theta_{,m},$$

i. e. the mass fluxes do not depend on the electric field.

Multiplying both sides of (4.22) by $z^{(l)}\mathcal{F}$, the charge current equations are obtained:

$$(4.24)\qquad \tilde{J}_k^{(l)} = \sigma_{km}^{(l)} E_m + \varkappa_{km}^{(l)} c_{,m}^{(l)} + w_{km}^{(l)} \theta_{,m},$$

where

$$\sigma_{km}^{(l)} = -\lambda_{km}^{(l)} z^{(l)} \mathcal{F} > 0$$

is the specific electric charge conductivity tensor of the l th kind,

$$\varkappa_{km}^{(l)} = \lambda_{km}^{(l)} N^{(ll)},$$

$$w_{km}^{(l)} = \lambda_{km}^{(ln+1)} + \lambda_{km}^{(ll)} H^{(l)}.$$

It is worth mentioning that transport equations of the form (4.24) are used for description of the electric properties of the semiconductors in the solid-state physics. These equations could be obtained by statistical considerations [17].

The obtained result (2.24) is in conformity with Bassett's point of view [7], namely that the electric properties of the bones are similar to those of the semiconductors.

Conclusion

The obtained constitutive equations together with the balance equations form a complete system, allowing simultaneous study of the processes of deformation, polarization, of the chemical reactions and of the transport phenomena.

It is accepted that in the existing up to now models, constructed entirely on the classical piezoelectric theory, the substance accumulated at the bone remodelling is determined only by the polarization on the bone surface [9], or by internal electric field.

In contrast, in the present paper it is taken into account the charge conductivity. The deformation of the bone causes generation of surface electric charges and of internal electric field. This leads to generation of internal currents and consequently to a new distribution of the charges. The positive charge carriers move towards the negative charged surfaces. After certain period of time the diffusion current becomes equal to the conductivity current, and there will be a new thermodynamical equilibrium, but the volume area near to the negative surfaces will be positively charged. It means that in the region near to the negative surfaces there will be increased concentration of cations and especially of Ca^{++}. On the other hand, it is well known that Ca^{++} is responsible for the cell mitosis [7]. Thus, it is expected that the bone will grow in these regions. The inverse prosess will be realized near to the positively charged surfaces.

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Электромеханические взаимодействия в физиологически мокрых костях

Н. Петров

(Резюме)

Целью работы является построение термодинамической модели с одновременным учетом процессов деформации, электрической поляризации, процессов транспорта и химических реакций. Полученные линейные конститутивные уравнения и уравнения сохранения предлагаются в качестве математической модели электромеханического взаимодействия в физиологически мокрых костях.

Результаты настоящей работы можно использовать для описания процессов повторного моделирования костей, подверженных механической нагрузке.