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DRAINAGE DISTANCE OF STRESS INDUCED FLUID FLOW IN CORTICAL BONE

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NOMENCLATURE

P - fluid pressure;

D - consolidation "diffusion" constant;

x, y - Cartesian coordinates in the plane defined by a transverse section through the osteons;

x^{HC}, y^{HC} - co-ordinates of the points within the area S^{HC} occupied by the Haversian Canals;

h - half of the specimen thickness;

S - uniaxial stress, acting in x direction;

μ_1 - shear modulus of bone matrix;

K_1 - bulk modulus of bone matrix;

K_w - bulk modulus of fluid.

1. Introduction

The problem of the effect of stress on fluid flow in a single osteon was modeled by using a finite element technique [1], and Biot's [2] single porosity consolidation theory. They succeeded in modeling the cusp-like shape of the streaming potential experimentally observed [3, 4]. The computational model does not explicitly incorporate either the canalicular - lacunar porosity or the microporosity of the osteon. Later, the theory was generalized [5] with more complicated microstructural assumptions for the canalicular annulus [6]. In order to describe qualitatively the macroscopic potentials in entire bone specimens, Salzstein et al., [7] developed a two-phase model for stress induced fluid flow in cortical bone. The underlying hypothesis of this model is that while the radial directed flows in osteons are responsible for the observed cusp shape of pressure relief in the osteons, the transcortical flow within the microporosity compartment causes the macroscopic pressure and streaming potential. This formulation was successful in describing the frequency dependence of the streaming potential magnitude and phase relative to load [8], but it is in conflict with later experimental findings [9, 10] in which they observed that the relaxation times were independent of specimen thickness. The Salzstein et. al. theory predicts, for the case of instantaneous loading, a quadratic relation for the above dependence, clearly in conflict with [9]. The same problem exists for model [6] which models the transcortical flow through continuous canalicular-lacunar pathways. Otter et al., and MacGinitie et al. explain their observation with the hypothesis that in multi-osteonal specimens, the principle fluid drainage resulting from applied stresses occurs to Haversian canals. This hypothesis for no fluid exchange between osteons was used, in literature, first [1], but the respective paper was, incorrectly cited and interpreted [11] as case of physically impenetrable cement line. In fact we explicitly postulated pressure extreme on the cement line as barriers for the fluid flow and we also remarked that it is not result of physical impenetrability of the cement line.

The interaction in a model system composed of six regularly positioned osteons, was computationally studied [11], using also Biot's single porosity consolidation theory for the case of cyclic loading [2]. They analyzed two cases, namely cement lines that were fully open and fully closed to fluid flow. Their results indicate that there is no fluid exchange between the osteons and consequently the drainage distance should be about the length of the osteon radius. However because in reality osteons are situated in bone randomly and it is not clear, a priori, to what extent Wang's conclusions are applicable for real cortical bone. More over, in the case of cyclic loading the drainage path is limited by the distance traveled by the fluid for the half time of the period of loading and reduces to zero as the frequency increases. For that reason, the relaxation process following the instantaneous loading is able more clearly to emulate, in the computational experiment, the fluid catchment areas.

The objective of the present study is to estimate the drainage distances in a more realistic anatomical computational model.

2. Anatomical and physical model

For the computational modeling of a system of osteons we mimic the anatomical structure of a real specimen used [3]. The anatomical model consists of 40 randomly situated osteons and cavities. The Haversian Canal radius is taken to be one sixth of the mean cement line radius, which is anatomically reasonable. In Fig. 1 open circles indicate the positions of the Haversian Canals. The solid line identifies the trajectory of the microelectrode in experiments [3]. The positions marked with black filled circles, numbered from 2 to 7 correspond to positions of osteon's cement lines. The black circles 1 and 8 correspond to the surface reference probes used to measure streaming potentials (pressure) at the specimen surfaces. In the present study we shall consider cement lines that are fully open to fluid flow. This idealization corresponds to the case of full osteon coupling considered by Wang et al., [11]. For the other case, namely impenetrable cement lines, it is a priori clear that the osteons must drain into Haversian Canals. For all intermediate model cases of semi penetrable cement lines the drainage distance should have values between this two limits.

Because of the complexity of the real anatomic structure it is reasonable to make the following simplifying assumptions as [1,11]:

1. The Haversian Canals and other cavities are considered as inclusions in a continuous isotropic medium.

2. The fluid influence on the deformed state of bone matrix is taken to be small.

3. The arterial fluid pressure in the Haversian Canal and other "macro" cavities is taken to be zero in respect to the stress induced pressure.

4. Micro stresses generated by inclusions in continuous media are located in closed areas around these inclusions and do not affect the drainage distance. That is why we eliminate them.

3. Mathematical model

According to the Biot's theory of consolidation [2] and the above simplifying assumption, the time dependence of the pressure P is described, for the case of instantaneous pure bending of the specimen, by the equations:

$$
(1)\qquad
\begin{aligned}
&\frac{\partial}{\partial t}P = D\left(\frac{\partial^2}{\partial x^2}P + \frac{\partial^2}{\partial y^2}P\right) \quad , \quad t > 0 \\
&P(x^{HC}, y^{HC}, t) = 0 \quad , \quad (x^{HC}, y^{HC}) \in S^{HC} \\
&\frac{\partial}{\partial y}P(x, \pm h, t) = 0 \quad , \quad t > 0 \\
&P(x, y, 0) = \vartheta\, y \quad , \quad 0 \le y \le h
\end{aligned}
\qquad ,
$$

where D is the consolidation "diffusion" constant, x, y are Cartesian coordinates in the plane defined by a transverse section through the osteons, x^{HC}, y^{HC} are the coordinates of points within the area occupied by the Haversian Canals, h is half of the specimen thickness. For determination of the constant ϑ we can use the following relation, validated for the initial pore fluid pressure in the paper [12]:

$$
(2)\qquad P(x, y, 0) = P^{cl}(y, 0) = P^{mp}(y, 0) = -\frac{K_w}{3K_1}\frac{4\mu_1 + 3K_1}{4\mu_1 + 3K_w}S \qquad ,
$$

where K_1, K_w and μ_1 are respectively bulk modulus of the bone matrix, bulk modulus of the fluid and shear modulus of the bone matrix. S is the uniaxial stress acting in the directuin of x, which in the case of pure bending, is a linear function of the coordinate y and inversely proportional to the bending radius R^b

$$
(3)\qquad S = \frac{3K_1\mu_1}{\mu_1 + K_1}\frac{y}{R^b}
$$

Therefore we have:

$$
(4)\qquad \vartheta = -\frac{K_w}{R^b}\frac{4\mu_1 + 3K_1}{4\mu_1 + 3K_w}\frac{\mu_1}{\mu_1 + K_1} \qquad .
$$

In the present paper we shall consider pressure relaxation at the marked positions, 2 to 7 in Fig.1 (positioned on cement lines), with respect to reference positions 1 and 8 (specimen surfaces). The scales for measuring, the time and the coordinates, as well as the values of the constants D and ϑ do not influence the comparative analysis of the relaxation times. To simplify the problem all variables taking part in equations (1), (2), (3), (4) would be reformulated in dimensionless form, which with respect to the decay times will change only the time scale but not the relative proportions. For the illustrative computational emulationof the problem the following reasonable relative values for the constants in (1): $h = 0.5$, $D = 1$, $\vartheta = -2$.

System (1) can be transformed into a system of ordinary differential equations using Finite Element Method (FEM) technique [13]. The respective matrix master equation of the problem is:

(5) $$[CM]\frac{d}{dt}\{P\} = [ST]\{P\} \quad ,$$

where $[CM]$ and $[ST]$ are the Mass and Stiffness matrices and the vector $\{P\}$ is composed of the nodal fluid pressure values.

To achieve the computational process, a regular grid of 3526 nodes and 6800 linear elements were used. To optimize PC resources (memory and computational time) the mass matrix $[CM]$ was condensed [14] and the stiffness matrix $[ST]$ was compressed by using the sparse matrix technique [15]. For the integration with respect to time, the absolutely stable, implicit differential scheme of Crank - Nickolson [13] was used.

4. Results and conclusions

When the bone strip is subjected to instantaneous pure bending, the initial fluid pressure distribution in the (x, y) plane of the specimen has the shape similar of the shape of the strain tensor trace. In the following moments of time the process of fluid flow and pressure relaxation appears. The formation of fluid catchment areas around the Haversian canals can be seen quite clearly on Fig.2. As it is predicted [1], at the cement lines, there are pressure extremes which block macroscopic flow. Such pressure extremes positioned over the cement lines have been experimentally observed in the case of homogeneous loading [3]. In the case of "pure bending", however, the observed streaming potential (pressure) "cusps" were not symmetric, and it was not possible to conclude the extent to which they could block macroscopic fluid flow.

The decay of pressure at the marked points on Fig. 1 is presented in Table 1 and on Fig. 3. As it can be seen, that independently of the position, the pressure at all marked positions decays to zero with the same decay constant, which means that the drainage distance is the same for all marked positions. This result strongly supports the hypotesis [1,9,10] that the fluid drains to Haversian canals rather than to the specimen surfaces. In spite of the local character of the fluid drainage the formed pressure profile emulated the stress generated potential profile, as measured [3]. An illustrative example is presented on Fig. 4 for the pressure distribution along the line which mimic the trajectory of the microelectrode positions in Starkebaum et al., [3].

The principle conclusions, following from the present computational emulation of the process of stress induced fluid flow in a system of osteons are:

- the formation of fluid catchment areas appear around the Haversian Canals;
- there is no objectivity for fluid exchange between the osteons;
- the fluid pressure decay is independent of the osteon position which suggest that in the cortical bone the drainage distance is equal to the osteon radius.

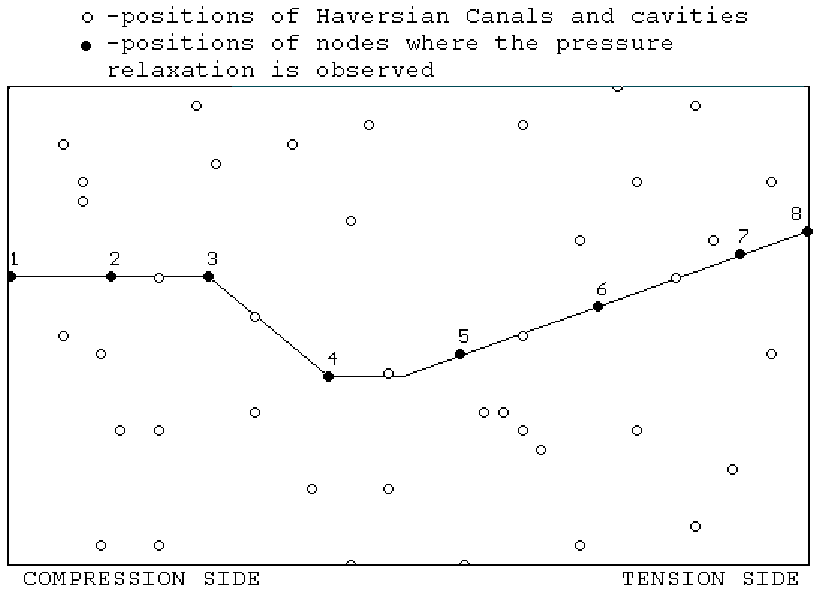

Fig. 1. Anatomical model of the bone specimen, as used in the experimental study [3].

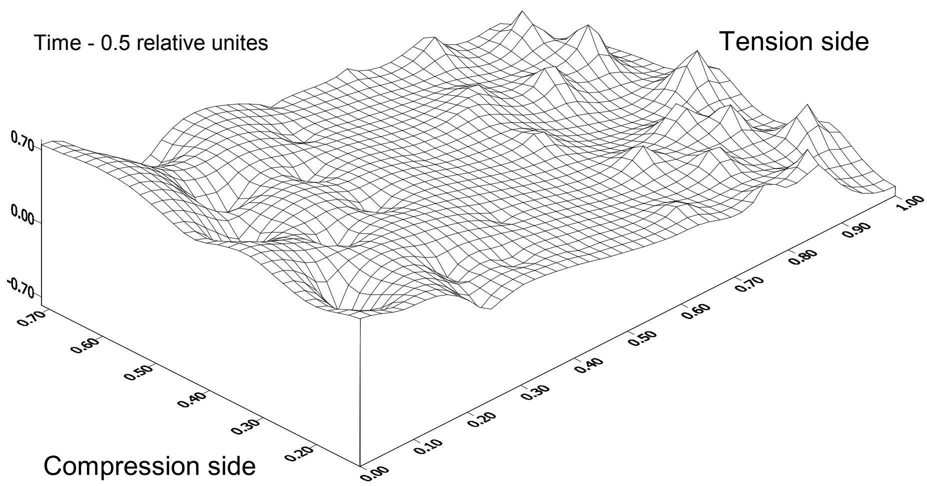

Fig. 2. The fluid catchment zones around Haversian canals predicted by FE model.

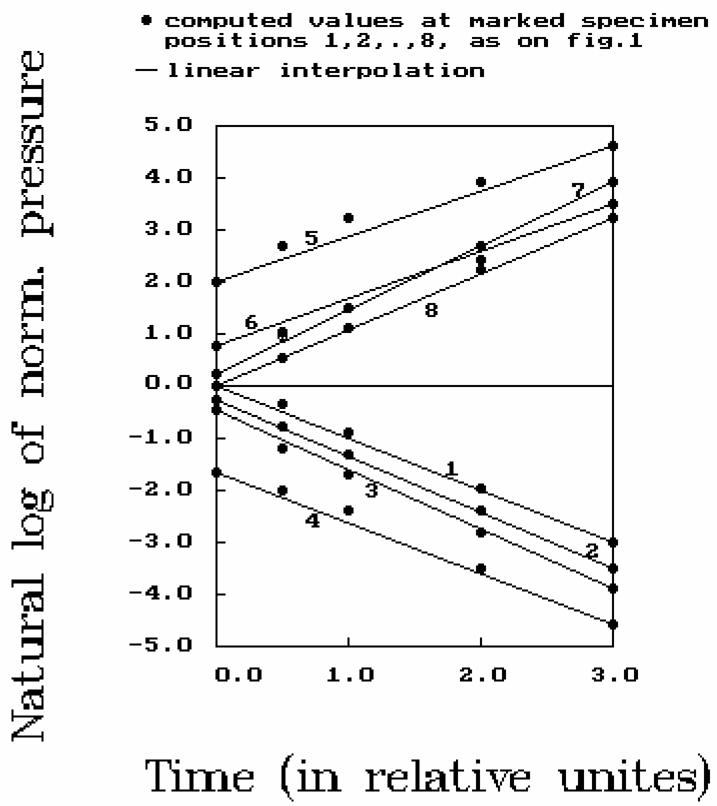

Fig. 3. Predicted normalized pressure versus time at the marked positions shown in Fig. 1.

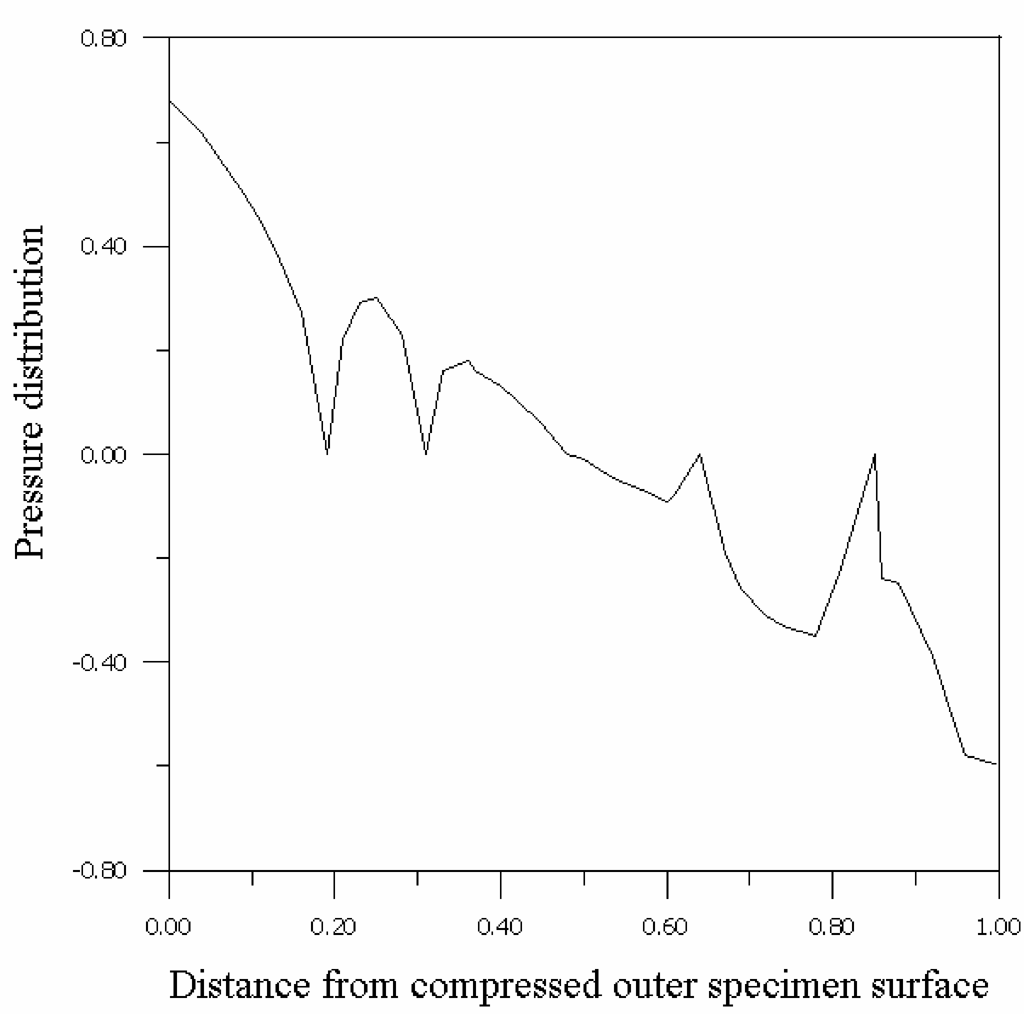

Fig. 4. The fluid pressure distribution along the trajectory of microelectrode probe in Starkebaum's experiment [3] predicted by FE model,.

Table 1

	Relative pressure $\frac{P(y,t)}{P(y,0)}$ decay							
	Position:							
Time	1	2	3	4	5	6	7	8
0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.1	0.94	0.96	0.94	1.0	0.92	0.99	0.81	0.93
0.5	0.69	0.60	0.63	0.73	0.51	0.76	0.48	0.60
1.0	0.41	0.34	0.37	0.45	0.29	0.48	0.27	0.33
2.0	0.14	0.11	0.12	0.16	0.11	0.18	0.09	0.11
3.0	0.05	0.05	0.04	0.05	0.05	0.07	0.03	0.04

* Time in relative unites

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